

Solving the 3x5 torus puzzle

Pensieve Header: Solving the 3x5 torus puzzle.

TheGenerators

```
n = 15;
g1 = Cycles[{{1, 6, 11}}];
g2 = Cycles[{{2, 7, 12}}];
g3 = Cycles[{{3, 8, 13}}];
g4 = Cycles[{{4, 9, 14}}];
g5 = Cycles[{{5, 10, 15}}];
g6 = Cycles[{{1, 2, 3, 4, 5}}];
g7 = Cycles[{{6, 7, 8, 9, 10}}];
g8 = Cycles[{{11, 12, 13, 14, 15}}];
```

TheProgram

```
Clear[σ];
σ ° τ := PermutationProduct[τ, σ];
Feed[Cycles[{}]] := Null;
Feed[τ_] := Module[{i, j, k, l},
  i = Min[PermutationSupport[τ]];
  j = PermutationReplace[i, τ];
  If[Head[σ[i, j]] === Cycles,
    Feed[InversePermutation[σ[i, j]] ° τ],
    (*Else*) σ[i, j] = τ;
  For[k = 1, k < n, ++k,
    For[l = k + 1, l ≤ n, ++l,
      If[Head[σ[k, l]] === Cycles,
        Feed[σ[i, j] ° σ[k, l]]; Feed[σ[k, l] ° σ[i, j]]
      ]
    ]
  ]];
$RecursionLimit = ∞;
```

TheRun

```
Table[Feed[gα];  $\prod_{i=1}^n (1 + \text{Count}[\text{Range}[n], j_ /; \text{Head}[\sigma[i, j]] == \text{Cycles}])$ , {α, 8}]
```

TheRun

```
{3, 9, 27, 81, 243, 653 837 184 000, 653 837 184 000, 653 837 184 000}
```

15!

```
1 307 674 368 000
```

15! / 2

```
653 837 184 000
```

```

ImageCrop[
  Rasterize[Graphics[{
    EdgeForm[Thin],
    Table[
      If[Head[ $\sigma_{i,j}$ ] === Cycles ||  $i = j$ ,
        {Red, Rectangle[{ $i - 1$ ,  $n - j$ }, { $i$ ,  $n - j + 1$ ]}}],
        {Yellow, Rectangle[{ $i - 1$ ,  $n - j$ }, { $i$ ,  $n - j + 1$ ]}}
      ], {i, n}, {j, i, n}
    ]
  }, AspectRatio  $\rightarrow$  1 / GoldenRatio, ImageSize  $\rightarrow$  720], RasterSize  $\rightarrow$  720]
]

```

