Penneys: Constructing subfactors with jellyfish

December-17-12
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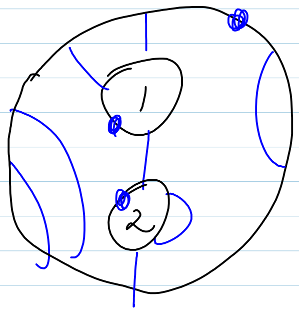
Subfactors $\rightleftarrows$ Planar Algebra

So we only talk about planar algebras...

<u>Def</u> planar algebra: seq (P_n) of v.s., along with an action by "planar tangles":

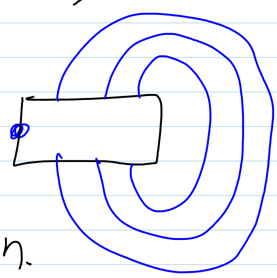


$: P_3 \otimes P_4 \longrightarrow P_9$

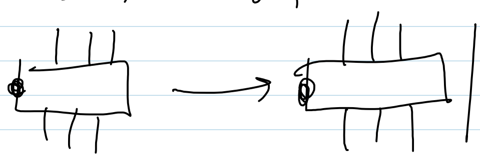
with an obvious composition property...

Example - Temperley-Lieb w/ $\delta > 0$

* TL_{2n} is an algebra.
* TL_{2n} has a trace:



* TL_{2n} has an "adjoint".
  - conjugate linear reflection.
* $\exists$ inclusion $TL_{2n} \hookrightarrow T_{2n+2}$:



<u>Thm</u> (Jones, 1983) $\langle x, y \rangle_{2n} = tr_{2n}(y^* x)$ is positive semi-definite for all n iff $\delta \in \{2\cos(\frac{\pi}{n}) : n \geq 3\} \cup [2, \infty)$

<u>Def</u> $P = (P_n)$ is a "factor" or "fantastic", if P is

1. evaluable: $\dim P_n < \infty$, $P_0 \cong \mathbb{C}$ under $\bigcirc \longrightarrow 1$.
2. P has an adjoint, and $\langle x, y \rangle_{2n}$

is positive definite for every n.

<u>Example</u> Tangles T_{2n} — not "fantastic".

<u>Example</u> Tensors. T_n = non-commutative monomials in some fixed involutive alphabet.

Fusion graph of P: (P fantastic)

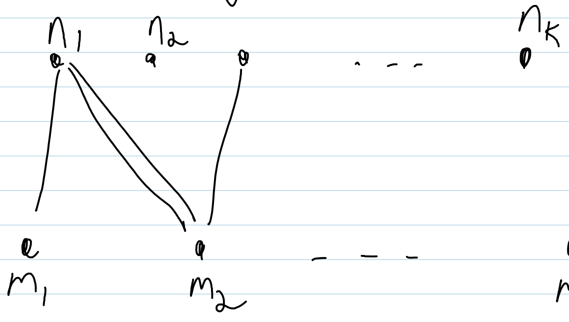
$$P_0 \hookrightarrow P_2 \hookrightarrow P_4 \hookrightarrow P_6 \hookrightarrow \cdots$$

P_{2n} is a complex s.s. algebra:

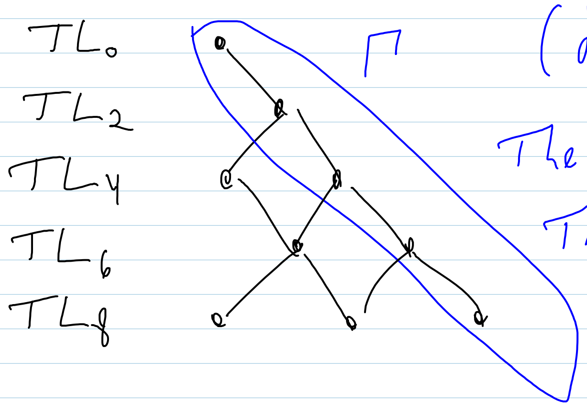
$$P_{2n} = \bigoplus_{i=1}^{k} M_{n_i}(\mathbb{C})$$

$$P_{2n+2} = \bigoplus_{j=1}^{\ell} M_{m_j}(\mathbb{C})$$

The Bratteli diagram:



For TL @ $\delta \geq 2$:



($\delta = \|\Gamma\|$)

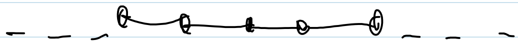
The fusion graph/
The principal graph –
The union of the new parts & edges connecting them.

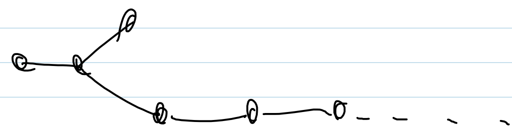
Q: What graphs may arise?

For $\delta<2$: ADE classification

For $\delta=2$: affine Dynkin diagrams,

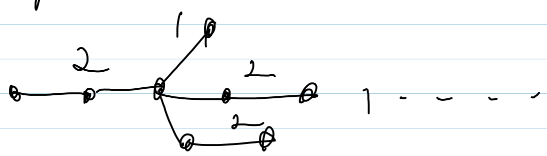
A_∞: 

$A_\infty^{(1)}$: 

D_∞: 

For $\delta>2$: ??

Some examples:



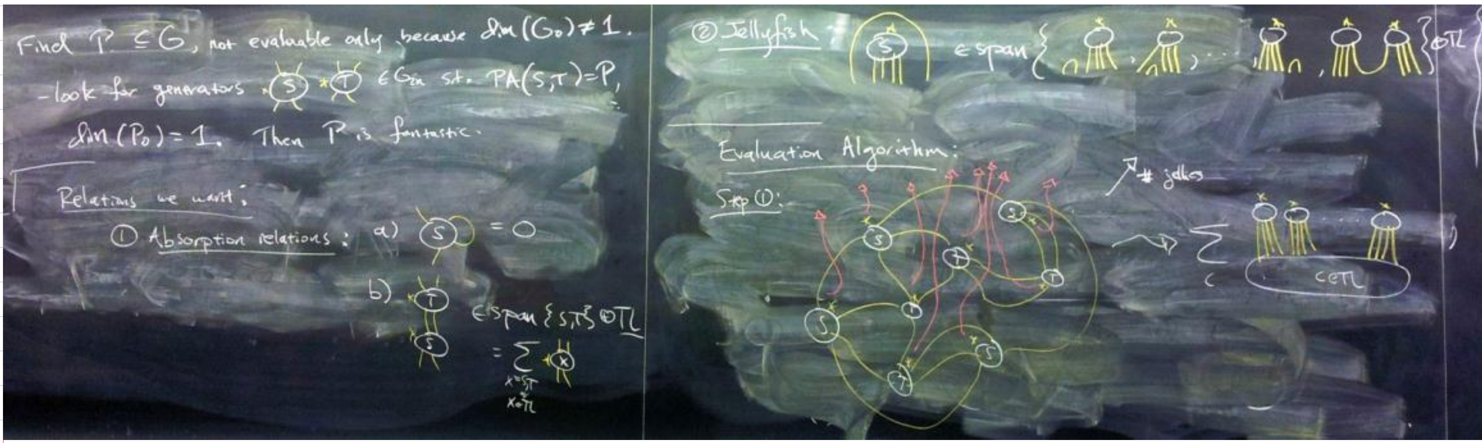
$\|\ \| = \sqrt{\frac{5+\sqrt{21}}{2}}$

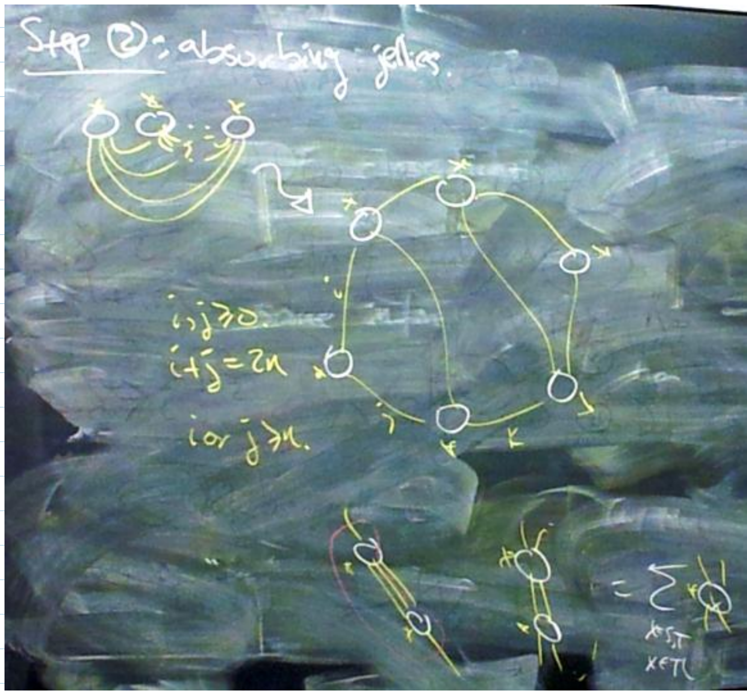
"2221"

Likewise there's "3333", "4442"

Thm (Bigelow–P.) 1. If P is a factor P.A., TFAE.

1. Γ is a "spoke graph"
2. Γ truncated 2 past the branch is a spoke graph.
3. P can be presented by generators & jelly-fish relations.


Find P ⊆ G, not evaluable only because dim(G₀) ≠ 1.
- look for generators S, T ∈ G_in s.t. PA(S,T) = P,
dim(P₀) = 1. Then P is fantastic.
Relations we want:
① Absorption relations: a)
= 0
b)
∈ span{S,T} ⊗ TL
② Jellyfish:
∈ span{
} ⊗ TL
Evaluation Algorithm:
Step ①:
jellies
c ∈ TL


Step ②: absorbing jellies.
i, j ≥ 0.
i + j = 2n
i or j ≥ n.