| Quantum topology (easy) | classical topology (hard & mysterious) |

Jones polynomial $J_L(q)$:

$$q J_{\nearrow}(q) - q^{-1} J_{\nearrow}(q) = (q^{1/2} - q^{-1/2}) J_{)(}(q)$$

$$J_\bigcirc(q) = q^{1/2} + q^{-1/2}$$

$$J_K(q) \in \mathbb{Z}[q^{\pm 1/2}]$$

Coloured Jones Poly....

<u>Definition</u> A sequence $F_n(q) \in \mathbb{Q}(q^{1/2})$ is
q-holonomic if it satisfies a linear recursion
relation with coefficients polynomials in
q and q^n.

----- The A-J conjecture.

In 2004: Aganagic-Vafa conjectured that one more variable can be added to this story. "the Homfly variable"

In 2007: Gukov + many conjectured you can add another variable for Khovanov homology.