Why J?

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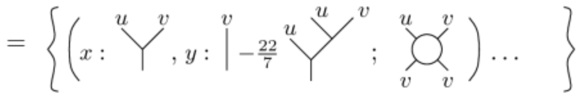
Continues 2012-08.

The Meta-Group-Action M. Let T be a set of "tail labels" ("balloon colours"), and H a set of "head labels" ("hoop colours"). Let $FL = FL(T)$ and $FA = FA(T)$ be the (completed graded) free Lie and free associative algebras on generators T and let $CW = CW(T)$ be the (completed graded) vector space of cyclic words on T, so there's $\mathrm{tr} : FA \to CW$. Let $M(T,H) := \{(\bar\lambda = (x : \lambda_x)_{x\in H}; \omega) : \lambda_x \in FL, \omega \in CW\}$



Operations. Set $(\bar\lambda_1;\omega_1) * (\bar\lambda_2;\omega_2) := (\bar\lambda_1 \cup \bar\lambda_2; \omega_1+\omega_2)$ and with $\mu = (\bar\lambda;\omega)$ define

$$tm^{uv}_w : \mu \mapsto \mu \mathbin{/\!/} (u, v \mapsto w),$$

$$hm^{xy}_z : \mu \mapsto \Big(\big(\ldots, \widehat{x : \lambda_x}, \widehat{y : \lambda_y}, \ldots, z : \mathrm{bch}(\lambda_x, \lambda_y)\big); \omega\Big)$$

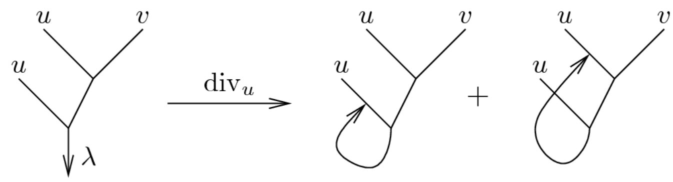
$$hta^{xu} : \mu \mapsto \underbrace{\overbrace{\mu \mathbin{/\!/\!/}}^{\text{“stable apply”}} (u \mapsto e^{\mathrm{ad}\,\lambda_x}(\bar u)) \mathbin{/\!/} (\bar u \mapsto u)}_{\mu /\!/ CC_u^{\lambda_x}} + \underbrace{(0; J_u(\lambda_x))}_{\text{the “}J\text{-spice”}}$$

The Meta-Cocycle J. Set $J_u(\lambda) := J(1)$ where

$$J(0) = 0, \qquad \lambda_s = \lambda \mathbin{/\!/} CC_u^{s\lambda},$$

$$\frac{dJ(s)}{ds} = (J(s) \mathbin{/\!/} \mathrm{der}(u \mapsto [\lambda_s, u])) + \mathrm{div}_u\,\lambda_s,$$

and where $\mathrm{div}_u\,\lambda := \mathrm{tr}(u\sigma_u(\lambda))$, $\sigma_u(v) := \delta_{uv}$, $\sigma_u([\lambda_1,\lambda_2]) := \iota(\lambda_1)\sigma_u(\lambda_2) - \iota(\lambda_2)\sigma_u(\lambda_1)$ and ι is the inclusion $FL \hookrightarrow FA$:



Claim. $CC_u^{\mathrm{bch}(\lambda_1,\lambda_2)} = CC_u^{\lambda_1} \mathbin{/\!/} CC_u^{\lambda_2 /\!/ CC_u^{\lambda_1}}$ and

$$J_u(\mathrm{bch}(\lambda_1,\lambda_2)) = J_u(\lambda_1) \mathbin{/\!/} CC_u^{\lambda_2 /\!/ CC_u^{\lambda_1}} + J_u(\lambda_2 \mathbin{/\!/} CC_u^{\lambda_1}),$$

and hence tm, hm, and hta form a meta-group-action.

Does it make sense to "turn on the interaction strength"?

"Euler only the active things"? 2012-04/ An Euler proof of BCH may be useful

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Most times I'm just wrong.

Lie-theoretically, it is weird that the subscript-input to J would be "a generator". I'd expect it to more generally be "an element", or "a derivation", or "a functional"....