

Current Support and Pending Support for Dror Bar-Natan.

I currently hold an NSERC Discovery grant #453990 titled "Knot Theory and Algebra", \$28,000 for each of five years 2008-2013, to a total of \$140,000.

I am currently applying for a further 5-year discovery grant and I expect to learn the status of my application around April 2013.

Balloons and Hoops and their Universal Finite-Type Invariant.
bⁿh

BF Theory, and an Ultimate Alexander Invariant
Dror Bar-Natan in Hamburg, August 2012
weβ := <http://www.math.toronto.edu/~drorbn/Talks/Hamburg-1208>

Scheme. • Balloons and hoops in \mathbb{R}^4 , algebraic structure and relations with 3D.
• An ansatz for a “homomorphic” invariant: computable, related to finite-type and to BF.
• Reduction to an “ultimate Alexander invariant”.

$\mathcal{K}^{bh}(m, n)$.

Examples.

$\epsilon_x: \pi \rightarrow$ (diagram)

$\epsilon_u: \pi \rightarrow$ (diagram)

ρ_{ux}^+ (diagram)

ρ_{ux}^- (diagram)

I mean business.

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S := 0x13, 41 0x13, 21 0x12, 41;
E := 0x // 0x01, 1, 13 // 0x14, 0, 01 //
0x01, 4, 41 // 0x18, 4, 41;
E1(12) := 0x, 0x // 0x010, 4, 11;
W, W := 0x0(0) 1 0;

-1(0x01) := 1, 1; -2 (0x0) := -3 (0x0);
-4 (0x00) := 43 (0x00) := 40 (0x00) := 4 (0x00);
-5 (0x000) := 110 (0x000) := 100 (0x000);
110 (0x000) := 110 (0x000); -1 (0x0000);
N(0) 50 12 (0); 0, -24 (0x0);
-40 (0x00) := 40 (0x00); -110 (0x000);
100 (0x000) := 100 (0x000) := 100 (0x000);
-1(4) 12 := 2 (0); -2 (0); 3 (0); 24 (0x0);
24 (0x0) := 60 (0x00) := 210 (0x00) := 40 (0x000);
110 (0x000) := 104 (0x000) := 110 (0x000);
104 (0x000) := 110 (0x000) := 110 (0x000);

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Meta-associativity.

$$m_a^{ab} \parallel m_b^{ac} = m_b^{bc} \parallel m_a^{ab}$$

Tangle concatenations $\rightarrow \pi_1 \times \pi_2$.

Thus we seek homomorphic invariants of \mathcal{K}^{bh} !

Invariant #0. With Π_1 denoting “honest π_1 ”, map $\gamma \in \mathcal{K}^{bh}(m, n)$ to the triple $(\Pi_1(\gamma^c), (u_i), (x_j))$, where the meridian of the balls u_i normally generate Π_1 , and the “longitudes” x_j are some elements of Π_1 .
* acts like *, tm acts by “merging” two meridians/generators, hm acts by multiplying two longitudes, and hta^{xu} acts by “conjugating a meridian by a longitude”:
 $(\Pi_1(u, \dots), (x, \dots)) \mapsto (\Pi_1(\bar{u})/(u = x\bar{u}x^{-1}), (\bar{u}, \dots), (x, \dots))$

Failure #0. Can we write the x 's as free words in the u 's?
If $x = uv$, compute $x \parallel hta^{xu}$:
 $x = uv \rightarrow \bar{u}v = u^xv = u^{uv}v = u^{u^xv}v = u^{u^{u^x}v}v = \dots$

The Meta-Group-Action M . Let T be a set of “tail labels” (“balloon colours”), and H a set of “head labels” (“hoop colours”). Let $FL = FL(T)$ and $FA = FA(T)$ be the (completed graded) free Lie and free associative algebras on generators T and let $CW = CW(T)$ be the (completed graded) vector space of cyclic words on T , so there's $\text{tr} : FA \rightarrow CW$. Let $M(T, H) := \{(\bar{\lambda} = (x : \lambda_x)_{x \in H}; \omega) : \lambda_x \in FL, \omega \in CW\}$

$$= \left\{ \left(x : \begin{array}{c} u & v \\ \diagdown & \diagup \\ & \end{array} y : \begin{array}{c} u & v \\ \diagdown & \diagup \\ & \end{array} \right) \dots \right\}$$

Operations. Set $(\bar{\lambda}_1; \omega_1) * (\bar{\lambda}_2; \omega_2) := (\bar{\lambda}_1 \cup \bar{\lambda}_2; \omega_1 + \omega_2)$ and with $\mu = (\bar{\lambda}; \omega)$ define

$$tm_w^{\mu} : \mu \mapsto \mu \parallel (u, v \mapsto w),$$

$$hm_x^{\mu} : \mu \mapsto \left((\dots, x : \bar{\lambda}_x, y : \bar{\lambda}_y, \dots, z : \text{bch}(\lambda_x, \lambda_y)) ; \omega \right)$$

“usable apply”

$$hta^{xu} : \mu \mapsto \mu \parallel \left(u \mapsto e^{ad \lambda_x}(\bar{u}) \parallel (\bar{u} \mapsto u) + (0; J_u(\lambda_x)) \right)$$

$\mu \parallel CC_u^\lambda$ the “J-spice”

A CC_u^λ example.

Operations Punctures & Cuts Connected Sums. $(\text{diagram}) * (\text{diagram}) = (\text{diagram})$

Meta-Group-Action. K : $K \parallel tm_w^{\mu}$: $K \parallel hm_x^{\mu}$: $K \parallel hta^{xu}$:

“MGA” (“//” is newspeak for “apply an operator” and for “composition left to right”)

Properties.

- Associativities: $m_a^{ab} \parallel m_a^{ac} = m_b^{bc} \parallel m_a^{ab}$, for $m = tm, hm$.
- Action axiom t : $tm_w^{\mu} \parallel hta^{xw} = hta^{xw} \parallel hta^{xu} \parallel tm_w^{\mu}$.
- Action axiom h : $hm_x^{\mu} \parallel hta^{xw} = hta^{xw} \parallel hta^{yu} \parallel hm_x^{\mu}$.
- SD Product: $dm_c^{ab} := hta^{ba} \parallel tm_c^{ab} \parallel hm_c^{ab}$ is associative.

Video and more at <http://www.math.toronto.edu/~drorbn/Talks/Hamburg-1208/>