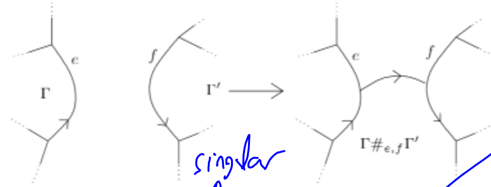
A: each may be either a transverse double point or merely a highlighted point on one of the strands, marked with the letter "F".

Given two graphs with selected edges (Γ, e) and (Γ', f), the *connected sum* of these graphs along the two chosen edges, denoted $\Gamma\#_{e,f}\Gamma'$, is obtained by joining e and f by a new edge. For this to be well-defined, we also need to specify the direction of the new edge, the cyclic orientations at each new vertex, and in the case of KTGs, the framing on the new edge. To compress notation, let us declare that the new edge be oriented from Γ towards Γ', have no twists, and, using the blackboard framing, be attached to the right side of e and f, as shown:



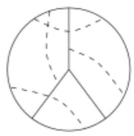
The classical way of introducing finite type invariants for links, A extended straightforwardly to KTGs, is to allow formal linear combinations of singular KTGs of the same skeleton, and to filter the resulting vector space by resolutions of singularities. An n-singular KTG is a trivalent graph immersed in R^3 with n ~~transverse double~~ points, A A resolution of an n- singular KTG with ~~n double points~~ is obtained ~~by~~ B replacing each double point by the difference of an over-crossing and an under-crossing, ~~which~~ This produces a linear combination of 2^n KTGs. The n-th piece of the finite type filtration $\mathcal{F}^n(\Gamma)$ is linearly generated by resolutions of of n-singular immersions of the skeleton Γ. Set

$$\mathcal{A}(\Gamma) = \bigoplus_{n=0}^{\infty} \mathcal{F}^n(\Gamma)/\mathcal{F}^{n+1}(\Gamma),$$

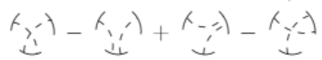
the associated graded space corresponding to the filtration. Finally, let $\mathcal{A} = \bigcup_\Gamma \mathcal{A}(\Gamma)$.

B: and each "F" point by the difference between the graph obtained by its removal & the graph obtained by its removal w/ one unit change to the framing (See Figure 7).

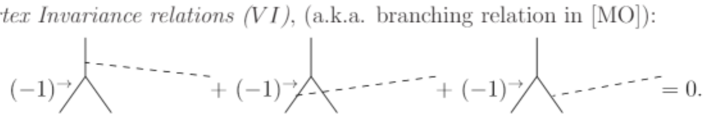
$\mathcal{A}(\Gamma)$ is best understood in terms of chord diagrams. A chord diagram of order n on a skeleton graph Γ is a combinatorial object consisting of a pairing of $2n$ points on the edges of Γ, up to orientation preserving homeomorphisms of the edges. Such a structure is illustrated by drawing n "chords" between the paired points, as seen in the figure on the right. As in the finite type theory of links, a chord represents the difference of an over-crossing and an under-crossing (i.e. a double point). This defines a map from chord diagrams to $\mathcal{A}(\Gamma)$, which is well defined and surjective.



There are two classes of relations that are in the kernel, the *Four Term relations (4T)*:

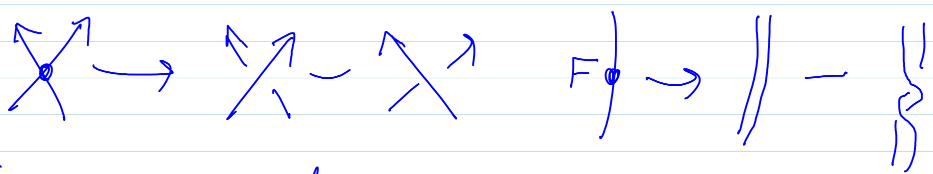


and the *Vertex Invariance relations (VI)*, (a.k.a. branching relation in [MO]):



In both pictures, there may be other chords in the parts of the graph not shown, but they have to be the same throughout. In 4T, all skeleton parts (solid lines) are oriented counterclockwise. In VI, the sign $(-1)^{\rightarrow}$ is -1 if the edge the chord is ending on is oriented to be outgoing from the vertex, and $+1$ if it is incoming.

Figure 7.



caption: The resolution of double points & "F" points.

(note that "F" points are needed because the resolution of  involves two units of framing change rather than 1).