BCH and div, II

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9:08 AM

From the A-T paper.

In order to discuss the groups KV_n and $\widehat{\mathrm{KV}}_2$ we introduce a Lie group 1-cocycle $j : \mathrm{TAut}_n \to \mathfrak{tr}_n$ which integrates the Lie algebra 1-cocycle $\mathrm{div} : \mathfrak{tder}_n \to \mathfrak{tr}_n$.

Proposition 5.1. *There is a unique map $j : \mathrm{TAut}_n \to \mathfrak{tr}_n$ which satisfies the group cocycle condition*

$$j(gh) = j(g) + g \cdot j(h), \tag{18}$$

That's also what I want for λ-calculus.

and has the property

nice language!

$$\frac{d}{ds} j(\exp(su))|_{s=0} = \mathrm{div}(u). \tag{19}$$

Proof. Let $\mathfrak{g}$ be a semi-direct sum of $\mathfrak{tder}_n$ and $\mathfrak{tr}_n$. The cocycle property of the divergence implies that the map $\mathfrak{tder}_n \to \mathfrak{g}$ defined by formula $u \mapsto u + \mathrm{div}(u)$ is a Lie algebra homomorphism. Define $j(\exp(u))$ by formula $\exp(u + \mathrm{div}(u)) = \exp(j(\exp(u)))\exp(u)$. For $g = \exp(u)$ and $h = \exp(v)$ we have

$$\exp(j(gh))gh = (\exp(j(g))g)(\exp(j(h))h) = \exp(j(g) + g \cdot j(h))gh$$

which implies (18).

Equations (18) and (19) imply the following differential equation for j:

$$\frac{d}{ds} j(\exp(su)) = \mathrm{div}(u) + u \cdot j(\exp(su)).$$

Given the initial condition $j(e) = 0$, this equation admits a unique solution,

$$j(\exp(u)) = \frac{e^u - 1}{u} \cdot \mathrm{div}(u)$$

which proves uniqueness of the cocycle j. □

There ought to be a direct proof that the j given by this formula satisfies (18).

In λ calculus: (approximate)

$$\left.\frac{d}{ds} j(su)\right|_{s=0} = \mathrm{div}_y u$$

$$j((s_1+s_2)u) = (s_1u)j(s_2u) + j(s_1u)$$

So

$$\frac{d}{ds} j(su) = u\,j(su) + \mathrm{div}_y u$$

Aside: solve $\dot{f} = uf + a$

Sol'n: $f = \dfrac{e^{tu}-1}{u}a$ ✓

$$\dot{f} = e^{tu} a$$

I must have a better description of StableApply?!

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There may not be one.