

Pensieve header: The free-Lie meta-group-action structure; continued 2012-08.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-07"];
<< "FreeLie.m"

Randomλ[d_, m_, n_] := Module[{bas},
  bas = AllLyndonWords[{d}, Characters[StringTake["1234567890", m]]];
  λ[
    MakeCWSeries[RandomInteger[{-2, 2}, Length[bas]].bas /. LW → CW],
    Sum[h[j] MakeLieSeries[RandomInteger[{-2, 2}, Length[bas]].bas], {j, n}]
  ]
];
hL[λ_] := Union[Cases[λ, h[s_] → s, Infinity]];
λ[ω_, μ_][d_] := λ[ω[d], μ /. s_LieSeries → s[d]];
λ[ω1_, μ1_] ≡ λ[ω2_, μ2_] :=
  ω1 ≡ ω2 && (And @@ ((D[μ1, h[#]] ≡ D[μ2, h[#]]) & /@ hL[{μ1, μ2}]));
LieDerivation[der_][λ[ω_, μ_]] := λ[ω // der, Collect[μ, _h, der]];
LieMorphism[mor_][λ[ω_, μ_]] := λ[ω // mor, Collect[μ, _h, mor]];

{Randomλ[1, 3, 3], Randomλ[3, 3, 3]}

{λ[CWS[CW[1] - CW[2] + CW[3], 0, 0], h[1] LS[-2 ⟨1⟩ - 2 ⟨3⟩, 0, 0] +
  h[2] LS[-2 ⟨1⟩ + 2 ⟨2⟩ + 2 ⟨3⟩, 0, 0] + h[3] LS[-2 ⟨1⟩ - ⟨2⟩ + 2 ⟨3⟩, 0, 0]],
  λ[CWS[CW[1] - CW[2] + 2 CW[3], -CW[13] - CW[23],
    2 CW[112] - 2 CW[113] + 2 CW[122] - CW[123] - 2 CW[132] + CW[133] + CW[223]],
  h[1] LS[-2 ⟨1⟩ + ⟨2⟩, 2 ⟨12⟩ + 2 ⟨13⟩ - 2 ⟨23⟩, 2 ⟨112⟩ - ⟨113⟩ + ⟨122⟩ - ⟨123⟩ +
    2 ⟨132⟩ + ⟨133⟩ + ⟨223⟩] + h[2] LS[-⟨1⟩ + ⟨3⟩, -⟨12⟩ + ⟨13⟩ + ⟨23⟩,
    ⟨112⟩ - 2 ⟨113⟩ + ⟨122⟩ - 2 ⟨123⟩ - 2 ⟨132⟩ + 2 ⟨133⟩ + ⟨233⟩] + h[3]
    LS[-⟨1⟩ - 2 ⟨2⟩ + ⟨3⟩, 2 ⟨12⟩ + 2 ⟨23⟩, -2 ⟨112⟩ + ⟨113⟩ + ⟨122⟩ + ⟨133⟩ - ⟨223⟩ - ⟨233⟩]]}

λ /: λ[ω1_, μ1_] λ[ω2_, μ2_] := λ[
  AddCWSeries[ω1, ω2],
  Collect[μ1 + μ2, _h,
    (# /. ls1_LieSeries + ls2_LieSeries → AddLieSeries[ls1, ls2]) &
  ]
];
tm[x_, y_, z_][λ[ω_, μ_]] := λ[ω, μ] // LieMorphism[{⟨x⟩ → ⟨z⟩, ⟨y⟩ → ⟨z⟩}];
hm[x_, y_, z_][λ[ω_, μ_]] := λ[ω, Plus[
  μ /. {h[x] → 0, h[y] → 0},
  h[z] BCH[D[μ, h[x]], D[μ, h[y]]]
]];
hta[x_, y_, z_][λ[ω_, μ_]] := Module[{μx, Adμx},
  μx = MakeLieSeries[D[μ, h[x]]];
  Adμx = LieMorphism[{LW[y] → Ad[ScaleLieSeries[-1, μx]][LW[z]]}];
  λ[
    AddCWSeries[StableApply[Adμx, ω], J[LW[y], μx]],
    Collect[μ, _h, StableApply[Adμx, #] &]
  ]
];
hta[x_, y_][λ[ω_, μ_]] :=
  λ[ω, μ] // hta[x, y, ⟨"z"⟩] // LieMorphism[{LW["z"] → LW[y]}];
dm[x_, y_, z_][λ_] := λ // hta[y, x] // tm[x, y, z] // hm[x, y, z];
Rp[x_, y_] := λ[MakeCWSeries[0], h[y] MakeLieSeries[⟨x⟩]];
Rm[x_, y_] := λ[MakeCWSeries[0], h[y] MakeLieSeries[-⟨x⟩]];

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n = $SeriesShowDegree = $SeriesCompareDegree = 4;
Print /@ {
  λ0 = Randomλ[3, 2, 2],
  λ0 // hta[1, 1],
  λ1 = ReplacePart[λ0, 1 → MakeCWSeries[0]],
  λ1 // hta[1, 1]
};

λ[CWS[-2 CW[1] + 2 CW[2], CW[12], -2 CW[122], 0],
h[1] LS[-⟨1⟩ - ⟨2⟩, ⟨12⟩, -⟨112⟩ + 2 ⟨122⟩, 0] +
h[2] LS[⟨1⟩ + ⟨2⟩, -2 ⟨12⟩, -2 ⟨112⟩ - ⟨122⟩, 0]]

λ[CWS[-3 CW[1] + 2 CW[2],  $\frac{CW[12]}{2}$ ,  $\frac{2 CW[112]}{3} + \frac{CW[122]}{3}$ ,
 $\frac{3 CW[1112]}{8} - \frac{5 CW[1122]}{4} + \frac{7 CW[1212]}{4} - \frac{9 CW[1222]}{8}$ ],
h[1] LS[-⟨1⟩ - ⟨2⟩, 0,  $-\frac{\langle 112 \rangle}{2} + \frac{5 \langle 122 \rangle}{2}$ ,  $-\frac{2 \langle 1112 \rangle}{3} + \frac{4 \langle 1122 \rangle}{3} + \frac{7 \langle 1222 \rangle}{3}$ ] +
h[2] LS[⟨1⟩ + ⟨2⟩, -⟨12⟩,  $-\frac{5 \langle 112 \rangle}{2} - \frac{5 \langle 122 \rangle}{2}$ ,  $\frac{2 \langle 1112 \rangle}{3} - \frac{23 \langle 1122 \rangle}{6} - \frac{11 \langle 1222 \rangle}{6}$ ]]

λ[CWS[0, 0, 0, 0], h[1] LS[-⟨1⟩ - ⟨2⟩, ⟨12⟩, -⟨112⟩ + 2 ⟨122⟩, 0] +
h[2] LS[⟨1⟩ + ⟨2⟩, -2 ⟨12⟩, -2 ⟨112⟩ - ⟨122⟩, 0]]

λ[CWS[-CW[1],  $-\frac{CW[12]}{2}$ ,  $\frac{2 CW[112]}{3} + \frac{7 CW[122]}{3}$ ,
 $\frac{3 CW[1112]}{8} - \frac{CW[1122]}{4} + \frac{3 CW[1212]}{4} - \frac{9 CW[1222]}{8}$ ],
h[1] LS[-⟨1⟩ - ⟨2⟩, 0,  $-\frac{\langle 112 \rangle}{2} + \frac{5 \langle 122 \rangle}{2}$ ,  $-\frac{2 \langle 1112 \rangle}{3} + \frac{4 \langle 1122 \rangle}{3} + \frac{7 \langle 1222 \rangle}{3}$ ] +
h[2] LS[⟨1⟩ + ⟨2⟩, -⟨12⟩,  $-\frac{5 \langle 112 \rangle}{2} - \frac{5 \langle 122 \rangle}{2}$ ,  $\frac{2 \langle 1112 \rangle}{3} - \frac{23 \langle 1122 \rangle}{6} - \frac{11 \langle 1222 \rangle}{6}$ ]]

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Testing tm

```

n = $SeriesShowDegree = $SeriesCompareDegree = 3;
Print /@ {λ0 = Randomλ[n, 4, 1],
  λ0 // tm[1, 2, 2],
  λ0 // tm[2, 3, 3],
  t1 = λ0 // tm[1, 2, 2] // tm[2, 3, 3],
  t2 = λ0 // tm[2, 3, 3] // tm[1, 3, 3],
  t1 ≡ t2
};

```

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λ[CWS[-2 CW[1] + 2 CW[3], 2 CW[12] + 2 CW[14] + 2 CW[23] + CW[24] + CW[34], -CW[112] +
  2 CW[113] + CW[114] + CW[122] + 2 CW[123] + CW[124] + 2 CW[132] - CW[134] - 2 CW[142] -
  2 CW[143] + CW[144] - 2 CW[223] + CW[233] + 2 CW[243] + CW[244] + 2 CW[334] - 2 CW[344]],
h[1] LS[-2 ⟨1⟩ + 2 ⟨2⟩ + 2 ⟨3⟩ + ⟨4⟩, 2 ⟨12⟩ + 2 ⟨13⟩ - ⟨14⟩ - 2 ⟨23⟩ + 2 ⟨24⟩ + ⟨34⟩,
  -⟨112⟩ - 2 ⟨113⟩ + 2 ⟨114⟩ - 2 ⟨122⟩ - 2 ⟨123⟩ + 2 ⟨124⟩ + ⟨132⟩ + ⟨133⟩ - 2 ⟨134⟩ +
  2 ⟨142⟩ + 2 ⟨143⟩ + ⟨223⟩ - ⟨233⟩ - 2 ⟨234⟩ + ⟨243⟩ - 2 ⟨244⟩ - 2 ⟨334⟩ - 2 ⟨344⟩]]

λ[CWS[-2 CW[2] + 2 CW[3], 2 CW[22] + 2 CW[23] + 3 CW[24] + CW[34],
  4 CW[223] + CW[233] - CW[234] + 2 CW[244] + 2 CW[334] - 2 CW[344]], h[1] LS[2 ⟨3⟩ + ⟨4⟩,
  ⟨24⟩ + ⟨34⟩, -4 ⟨223⟩ + 2 ⟨224⟩ - 4 ⟨234⟩ + 3 ⟨243⟩ - 2 ⟨244⟩ - 2 ⟨334⟩ - 2 ⟨344⟩]]

λ[CWS[-2 CW[1] + 2 CW[3], 2 CW[13] + 2 CW[14] + 2 CW[33] + 2 CW[34],
  CW[113] + CW[114] + 5 CW[133] - 4 CW[143] + CW[144] - CW[333] + 4 CW[334] - CW[344]],
h[1] LS[-2 ⟨1⟩ + 4 ⟨3⟩ + ⟨4⟩, 4 ⟨13⟩ - ⟨14⟩ + 3 ⟨34⟩,
  -3 ⟨113⟩ + 2 ⟨114⟩ + 4 ⟨143⟩ - 5 ⟨334⟩ - 4 ⟨344⟩]]

λ[CWS[0, 4 CW[33] + 4 CW[34], 5 CW[333] + CW[334]],
h[1] LS[2 ⟨3⟩ + ⟨4⟩, 2 ⟨34⟩, -7 ⟨334⟩ - 4 ⟨344⟩]]

λ[CWS[0, 4 CW[33] + 4 CW[34], 5 CW[333] + CW[334]],
h[1] LS[2 ⟨3⟩ + ⟨4⟩, 2 ⟨34⟩, -7 ⟨334⟩ - 4 ⟨344⟩]]

True

λ[CWS[CW[2] + 2 CW[3] + CW[4], 2 CW[22] - 2 CW[23] + CW[24] + 2 CW[34],
  -2 CW[222] + CW[223] + 3 CW[224] + 2 CW[234] - CW[243] - 4 CW[244] - 2 CW[344]],
h[1] LS[-2 ⟨2⟩ - ⟨3⟩ - ⟨4⟩, -⟨24⟩ + ⟨34⟩,
  -2 ⟨223⟩ - 3 ⟨224⟩ - ⟨233⟩ - 4 ⟨234⟩ + 3 ⟨243⟩ - ⟨244⟩ - 2 ⟨334⟩ - ⟨344⟩]]

λ[CWS[3 CW[3] + CW[4], 2 CW[13] + 2 CW[14] - 2 CW[33] + CW[34],
  -CW[113] + 3 CW[133] + 2 CW[134] + 3 CW[143] - 2 CW[144] - 3 CW[333] - CW[334] - 4 CW[344]],
h[1] LS[-3 ⟨3⟩ - ⟨4⟩, -2 ⟨14⟩ + 2 ⟨34⟩,
  ⟨113⟩ - ⟨114⟩ - 2 ⟨133⟩ - 4 ⟨134⟩ + 3 ⟨143⟩ + ⟨144⟩ - 4 ⟨334⟩ - 3 ⟨344⟩]]

λ[CWS[3 CW[3] + CW[4], 3 CW[34], -CW[333] + 4 CW[334] - 6 CW[344]],
h[1] LS[-3 ⟨3⟩ - ⟨4⟩, 0, -12 ⟨334⟩ - 2 ⟨344⟩]]

λ[CWS[3 CW[3] + CW[4], 3 CW[34], -CW[333] + 4 CW[334] - 6 CW[344]],
h[1] LS[-3 ⟨3⟩ - ⟨4⟩, 0, -12 ⟨334⟩ - 2 ⟨344⟩]]

True

t1 = λ0 // tm[1, 2, 2] // tm[2, 3, 3] // InputForm

λ[LieSeries[LieMorphismOnLieSeries$110], h[1]*LieSeries[LieMorphismOnLieSeries$112]]

```

Testing hm

```

Print /@ {λ0 = Randomλ[4, 2, 3],
  λ0 // hm[1, 2, 2],
  t1 = λ0 // hm[1, 2, 2] // hm[2, 3, 3],
  t2 = λ0 // hm[2, 3, 3] // hm[1, 3, 3],
  t1 ≡ t2
};

```

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λ[CWS[CW[1] + CW[2], CW[12], CW[112] + CW[122]], h[1] LS[2 ⟨1⟩ + ⟨2⟩, ⟨12⟩, ⟨112⟩ + 2 ⟨122⟩] +
  h[2] LS[-2 ⟨1⟩ + 2 ⟨2⟩, ⟨12⟩, 2 ⟨112⟩] + h[3] LS[-⟨1⟩ - ⟨2⟩, 0, 0]]
λ[CWS[CW[1] + CW[2], CW[12], CW[112] + CW[122]],
  h[2] LS[3 ⟨2⟩, 5 ⟨12⟩, 7 ⟨112⟩ + 3 ⟨122⟩] + h[3] LS[-⟨1⟩ - ⟨2⟩, 0, 0]]
λ[CWS[CW[1] + CW[2], CW[12], CW[112] + CW[122]],
  h[3] LS[-⟨1⟩ + 2 ⟨2⟩,  $\frac{13 \langle 12 \rangle}{2}$ ,  $\frac{39 \langle 112 \rangle}{4} - \frac{\langle 122 \rangle}{2}$ ]]
λ[CWS[CW[1] + CW[2], CW[12], CW[112] + CW[122]],
  h[3] LS[-⟨1⟩ + 2 ⟨2⟩,  $\frac{13 \langle 12 \rangle}{2}$ ,  $\frac{39 \langle 112 \rangle}{4} - \frac{\langle 122 \rangle}{2}$ ]]
True

```

Testing hta

```

n = $SeriesShowDegree = $SeriesCompareDegree = 4;
Print /@ {λ0 = Randomλ[3, 2, 2], λ0 // hta[1, 1]};
λ[CWS[2 CW[1] - 2 CW[2], -2 CW[12], CW[112] - 2 CW[122], 0],
  h[1] LS[-⟨1⟩ + 2 ⟨2⟩, 2 ⟨12⟩, 2 ⟨112⟩ - 2 ⟨122⟩, 0] + h[2] LS[-⟨1⟩, ⟨12⟩, -2 ⟨112⟩ + ⟨122⟩, 0]]
λ[CWS[2 CW[1] - 2 CW[2], -2 CW[12], CW[112] - 2 CW[122], 12 CW[1122] - 12 CW[1212]],
  h[1] LS[-⟨1⟩ + 2 ⟨2⟩, 4 ⟨12⟩, 5 ⟨112⟩ - 8 ⟨122⟩,  $\frac{10 \langle 1112 \rangle}{3} - \frac{62 \langle 1122 \rangle}{3} + \frac{28 \langle 1222 \rangle}{3}$ ] +
  h[2] LS[-⟨1⟩, 3 ⟨12⟩, ⟨112⟩ - 3 ⟨122⟩,  $\frac{10 \langle 1112 \rangle}{3} - \frac{29 \langle 1122 \rangle}{3} + \frac{4 \langle 1222 \rangle}{3}$ ]]]
Print /@ {λ0 = Randomλ[n, 3, 2],
  t1 = λ0 // hta[1, 1] // hta[1, 2] // tm[1, 2, 1],
  t2 = λ0 // tm[1, 2, 1] // hta[1, 1],
  t1 ≡ t2
};

```

```

λ[CWS[-CW[1] + CW[2] - CW[3], -CW[12] + CW[13] - 2 CW[23],
  CW[113] - CW[122] - 2 CW[123] - 2 CW[132] - 2 CW[223] + CW[233],
  2 CW[1112] - 2 CW[1113] + CW[1122] - CW[1123] + 2 CW[1132] +
  2 CW[1133] - 2 CW[1213] + 2 CW[1222] + 2 CW[1223] + 2 CW[1232] + CW[1322] +
  2 CW[1323] - CW[1332] - 2 CW[1333] + 2 CW[2223] + 2 CW[2233] + CW[2333]],
h[1] LS[⟨2⟩ + 2 ⟨3⟩, -⟨12⟩ + ⟨13⟩ - ⟨23⟩, 2 ⟨112⟩ + ⟨113⟩ - ⟨122⟩ + 2 ⟨123⟩ + 2 ⟨132⟩ -
  ⟨133⟩ + 2 ⟨223⟩, -2 ⟨1112⟩ - 2 ⟨1113⟩ + 2 ⟨1122⟩ + 2 ⟨1123⟩ + 2 ⟨1132⟩ -
  2 ⟨1133⟩ - ⟨1213⟩ - 2 ⟨1223⟩ - ⟨1233⟩ - ⟨1332⟩ + 2 ⟨1333⟩ + ⟨2223⟩ + 2 ⟨2233⟩] +
h[2] LS[0, 2 ⟨12⟩, ⟨112⟩ - ⟨113⟩ - ⟨122⟩ - ⟨123⟩ - ⟨132⟩ - ⟨133⟩ + 2 ⟨223⟩ - 2 ⟨233⟩,
  2 ⟨1113⟩ - 2 ⟨1123⟩ - 2 ⟨1132⟩ + ⟨1133⟩ - ⟨1222⟩ -
  ⟨1232⟩ - 2 ⟨1233⟩ - 2 ⟨1322⟩ + ⟨1323⟩ - 2 ⟨1333⟩ - ⟨2333>]]

λ[CWS[-CW[3], -CW[11] - CW[13], -CW[111] - 5 CW[113] + CW[133],
  5 CW[1111] + 4 CW[1113] + CW[1133] + 4 CW[1313] - CW[1333]],
h[1] LS[⟨1⟩ + 2 ⟨3⟩, -2 ⟨13⟩, 4 ⟨113⟩ + ⟨133⟩, - $\frac{19 \langle 1113 \rangle}{3}$  -  $\frac{23 \langle 1133 \rangle}{3}$  +  $\frac{8 \langle 1333 \rangle}{3}$ ] +
h[2] LS[0, 0, ⟨113⟩ - 3 ⟨133⟩, ⟨1113⟩ - 3 ⟨1133⟩ + 3 ⟨1333>]]

λ[CWS[-CW[3], -CW[11] - CW[13], -CW[111] - 5 CW[113] + CW[133],
  5 CW[1111] + 4 CW[1113] + CW[1133] + 4 CW[1313] - CW[1333]],
h[1] LS[⟨1⟩ + 2 ⟨3⟩, -2 ⟨13⟩, 4 ⟨113⟩ + ⟨133⟩, - $\frac{19 \langle 1113 \rangle}{3}$  -  $\frac{23 \langle 1133 \rangle}{3}$  +  $\frac{8 \langle 1333 \rangle}{3}$ ] +
h[2] LS[0, 0, ⟨113⟩ - 3 ⟨133⟩, ⟨1113⟩ - 3 ⟨1133⟩ + 3 ⟨1333>]]

True

n = $SeriesShowDegree = $SeriesCompareDegree = 5;
Print /@ {λ0 = ReplacePart[Randomλ[n, 2, 3], 1 → MakeCWSeries[0]],
  t1 = λ0 // hta[1, 1] // hta[2, 1] // hm[1, 2, 1],
  t2 = λ0 // hm[1, 2, 1] // hta[1, 1],
  t1 ≡ t2
};

```

$$\lambda[\text{CWS}[0, 0, 0, 0, 0], \text{h}[1] \text{LS}[\langle 1 \rangle, \langle 12 \rangle, 2 \langle 112 \rangle - \langle 122 \rangle, \langle 1112 \rangle + \langle 1122 \rangle, -\langle 11112 \rangle - \langle 11122 \rangle - \langle 11212 \rangle - 2 \langle 11222 \rangle + \langle 12122 \rangle + \langle 12222 \rangle] + \text{h}[2] \text{LS}[2 \langle 1 \rangle, 0, -\langle 112 \rangle, -\langle 1112 \rangle + 2 \langle 1122 \rangle + \langle 1222 \rangle, 2 \langle 11112 \rangle + \langle 11122 \rangle + \langle 11212 \rangle + 2 \langle 11222 \rangle + 2 \langle 12222 \rangle] + \text{h}[3] \text{LS}[\langle 1 \rangle + 2 \langle 2 \rangle, 0, -2 \langle 112 \rangle + \langle 122 \rangle, \langle 1122 \rangle, \langle 11112 \rangle - 2 \langle 11122 \rangle + 2 \langle 11222 \rangle - 2 \langle 12122 \rangle]]]$$

$$\lambda\left[\text{CWS}\left[3 \text{CW}[1], -\text{CW}[12], \frac{3 \text{CW}[112]}{2} - \text{CW}[122], \frac{5 \text{CW}[1112]}{6} - 4 \text{CW}[1122] + 9 \text{CW}[1212] - \text{CW}[1222], -\frac{59 \text{CW}[11112]}{24} + \frac{23 \text{CW}[11122]}{6} - 10 \text{CW}[11212] + 5 \text{CW}[11222] - 5 \text{CW}[12122] + 3 \text{CW}[12222]\right], \text{h}[3] \text{LS}\left[\langle 1 \rangle + 2 \langle 2 \rangle, 0, -3 \langle 112 \rangle + \langle 122 \rangle, \frac{3 \langle 1112 \rangle}{2} + 2 \langle 1122 \rangle, \frac{11 \langle 11112 \rangle}{6} - \frac{9 \langle 11122 \rangle}{2} + \frac{5 \langle 11212 \rangle}{2} - 3 \langle 12122 \rangle\right] + \text{h}[1] \text{LS}\left[3 \langle 1 \rangle, \langle 12 \rangle, -3 \langle 112 \rangle - \langle 122 \rangle, \frac{13 \langle 1112 \rangle}{6} + 6 \langle 1122 \rangle + \langle 1222 \rangle, \frac{31 \langle 11112 \rangle}{12} - \frac{73 \langle 11122 \rangle}{6} + \frac{2 \langle 11212 \rangle}{3} - \frac{\langle 11222 \rangle}{2} + 3 \langle 12122 \rangle + 3 \langle 12222 \rangle\right]\right]$$

$$\lambda\left[\text{CWS}\left[3 \text{CW}[1], -\text{CW}[12], \frac{3 \text{CW}[112]}{2} - \text{CW}[122], \frac{5 \text{CW}[1112]}{6} - 4 \text{CW}[1122] + 9 \text{CW}[1212] - \text{CW}[1222], -\frac{59 \text{CW}[11112]}{24} + \frac{23 \text{CW}[11122]}{6} - 10 \text{CW}[11212] + 5 \text{CW}[11222] - 5 \text{CW}[12122] + 3 \text{CW}[12222]\right], \text{h}[1] \text{LS}\left[3 \langle 1 \rangle, \langle 12 \rangle, -3 \langle 112 \rangle - \langle 122 \rangle, \frac{13 \langle 1112 \rangle}{6} + 6 \langle 1122 \rangle + \langle 1222 \rangle, \frac{31 \langle 11112 \rangle}{12} - \frac{73 \langle 11122 \rangle}{6} + \frac{2 \langle 11212 \rangle}{3} - \frac{\langle 11222 \rangle}{2} + 3 \langle 12122 \rangle + 3 \langle 12222 \rangle\right] + \text{h}[3] \text{LS}\left[\langle 1 \rangle + 2 \langle 2 \rangle, 0, -3 \langle 112 \rangle + \langle 122 \rangle, \frac{3 \langle 1112 \rangle}{2} + 2 \langle 1122 \rangle, \frac{11 \langle 11112 \rangle}{6} - \frac{9 \langle 11122 \rangle}{2} + \frac{5 \langle 11212 \rangle}{2} - 3 \langle 12122 \rangle\right]\right]$$

True

Testing dm

```

SeriesShowDegree = 3;
SeriesCompareDegree = n = 5;
Timing[Print /@ {λ0 = Randomλ[n, 4, 4],
  t1 = λ0 // dm[1, 2, 1] // dm[1, 3, 1],
  t2 = λ0 // dm[2, 3, 2] // dm[1, 2, 1],
  t1 ≡ t2
};]

```

```

λ[CWS[-2 CW[1] - 2 CW[2] + CW[3] - CW[4], CW[12] - 2 CW[13] - CW[24],
  -CW[112] - CW[113] - CW[114] - CW[122] - 2 CW[124] + CW[132] + CW[133] + CW[134] -
  2 CW[142] - 2 CW[143] + CW[144] - 2 CW[223] + 2 CW[233] + CW[243] + 2 CW[334] - 2 CW[344]],
h[1] LS[2 ⟨4⟩, ⟨12⟩ + 2 ⟨13⟩ + ⟨23⟩ + 2 ⟨24⟩ + 2 ⟨34⟩,
  -⟨113⟩ + ⟨122⟩ - 2 ⟨123⟩ + ⟨124⟩ - 2 ⟨132⟩ - 2 ⟨133⟩ - 2 ⟨143⟩ + ⟨144⟩ +
  2 ⟨223⟩ - 2 ⟨224⟩ + 2 ⟨233⟩ - ⟨234⟩ + ⟨243⟩ - 2 ⟨244⟩ - ⟨334⟩ + 2 ⟨344⟩] +
h[2] LS[⟨1⟩ - ⟨2⟩ + 2 ⟨3⟩ - ⟨4⟩, -2 ⟨13⟩ - 2 ⟨14⟩ - ⟨23⟩ - 2 ⟨34⟩,
  -2 ⟨112⟩ + 2 ⟨113⟩ - 2 ⟨114⟩ - 2 ⟨122⟩ - ⟨123⟩ + 2 ⟨124⟩ + ⟨133⟩ - ⟨143⟩ +
  2 ⟨144⟩ + ⟨223⟩ + 2 ⟨224⟩ - ⟨233⟩ - ⟨234⟩ - ⟨243⟩ + 2 ⟨244⟩ + ⟨334⟩] +
h[3] LS[⟨1⟩ - 2 ⟨2⟩ - ⟨3⟩ + 2 ⟨4⟩, -2 ⟨12⟩ - ⟨13⟩ - ⟨23⟩ + 2 ⟨24⟩ - ⟨34⟩,
  ⟨113⟩ - ⟨114⟩ + 2 ⟨122⟩ - ⟨133⟩ - 2 ⟨134⟩ + 2 ⟨142⟩ - 2 ⟨143⟩ -
  2 ⟨144⟩ - ⟨223⟩ + 2 ⟨234⟩ - 2 ⟨243⟩ - 2 ⟨244⟩ - ⟨334⟩ + ⟨344⟩] +
h[4] LS[⟨1⟩ - ⟨4⟩, -2 ⟨12⟩ + ⟨13⟩ + ⟨14⟩ + 2 ⟨23⟩ - ⟨34⟩, -⟨112⟩ - 2 ⟨113⟩ - ⟨114⟩ - 2 ⟨124⟩ -
  2 ⟨132⟩ - ⟨134⟩ - 2 ⟨142⟩ - ⟨224⟩ + 2 ⟨233⟩ + 2 ⟨234⟩ - 2 ⟨243⟩ - ⟨244⟩ + ⟨334⟩ - 2 ⟨344⟩]]

λ[CWS[-3 CW[1] - CW[4], 6 CW[11] -  $\frac{7 CW[14]}{2}$ , - $\frac{35 CW[114]}{6}$  - 7 CW[144]],
h[4] LS[⟨1⟩ - ⟨4⟩, -⟨14⟩, 7 ⟨114⟩ -  $\frac{7 \langle 144 \rangle}{2}$ ] + h[1] LS[3 ⟨4⟩, 6 ⟨14⟩,  $\frac{11 \langle 114 \rangle}{3}$  -  $\frac{19 \langle 144 \rangle}{2}$ ]]

λ[CWS[-3 CW[1] - CW[4], 6 CW[11] -  $\frac{7 CW[14]}{2}$ , - $\frac{35 CW[114]}{6}$  - 7 CW[144]],
h[4] LS[⟨1⟩ - ⟨4⟩, -⟨14⟩, 7 ⟨114⟩ -  $\frac{7 \langle 144 \rangle}{2}$ ] + h[1] LS[3 ⟨4⟩, 6 ⟨14⟩,  $\frac{11 \langle 114 \rangle}{3}$  -  $\frac{19 \langle 144 \rangle}{2}$ ]]

True
{51.621, Null}

```

Testing R-Moves

```

(Rp[1, 2] Rm[3, 4]) // dm[1, 3, 1] // dm[2, 4, 2]
λ[CWS[0, 0, 0], h[1] LS[0, 0, 0] + h[2] LS[0, 0, 0]]
(Rp[1, 2] Rm[3, 4]) // dm[1, 3, 1] // dm[4, 2, 2]
λ[CWS[0, 0, 0], h[1] LS[0, 0, 0] + h[2] LS[0, 0, 0]]
t1 = Rp[1, 2] Rp[4, 3] Rp[5, 6] // dm[1, 4, 1] // dm[2, 5, 2] // dm[3, 6, 3]
λ[CWS[0, 0, 0],
  h[1] LS[0, 0, 0] + h[2] LS[⟨1⟩, 0, 0] + h[3] LS[⟨1⟩ + ⟨2⟩,  $\frac{\langle 12 \rangle}{2}$ ,  $\frac{\langle 112 \rangle}{12}$  +  $\frac{\langle 122 \rangle}{12}$ ]]
t2 = Rp[2, 3] Rp[1, 4] Rp[5, 6] // dm[1, 5, 1] // dm[2, 6, 2] // dm[3, 4, 3]
λ[CWS[0, 0, 0],
  h[1] LS[0, 0, 0] + h[2] LS[⟨1⟩, 0, 0] + h[3] LS[⟨1⟩ + ⟨2⟩,  $\frac{\langle 12 \rangle}{2}$ ,  $\frac{\langle 112 \rangle}{12}$  +  $\frac{\langle 122 \rangle}{12}$ ]]

```

`{t1[7], t2[7], t1[7] == t2[7]}`

$$\left\{ \lambda \left[0, h[3] \left(\frac{\langle 11111112 \rangle}{30240} - \frac{\langle 11111122 \rangle}{5040} + \frac{\langle 1111212 \rangle}{10080} + \frac{\langle 1111222 \rangle}{3780} + \frac{\langle 1112112 \rangle}{10080} + \frac{\langle 1112122 \rangle}{1680} + \frac{\langle 1112212 \rangle}{1260} + \frac{\langle 1112222 \rangle}{3780} + \frac{\langle 1121122 \rangle}{2016} - \frac{\langle 1121212 \rangle}{5040} + \frac{13 \langle 1121222 \rangle}{15120} + \frac{\langle 1122122 \rangle}{10080} - \frac{\langle 1122212 \rangle}{1512} - \frac{\langle 1122222 \rangle}{5040} + \frac{\langle 1212122 \rangle}{1260} - \frac{\langle 1212222 \rangle}{2016} - \frac{\langle 1221222 \rangle}{5040} + \frac{\langle 1222222 \rangle}{30240} \right) \right], \lambda \left[0, h[3] \left(\frac{\langle 11111112 \rangle}{30240} - \frac{\langle 11111122 \rangle}{5040} + \frac{\langle 1111212 \rangle}{10080} + \frac{\langle 1111222 \rangle}{3780} + \frac{\langle 1112112 \rangle}{10080} + \frac{\langle 1112122 \rangle}{1680} + \frac{\langle 1112212 \rangle}{1260} + \frac{\langle 1112222 \rangle}{3780} + \frac{\langle 1121122 \rangle}{2016} - \frac{\langle 1121212 \rangle}{5040} + \frac{13 \langle 1121222 \rangle}{15120} + \frac{\langle 1122122 \rangle}{10080} - \frac{\langle 1122212 \rangle}{1512} - \frac{\langle 1122222 \rangle}{5040} + \frac{\langle 1212122 \rangle}{1260} - \frac{\langle 1212222 \rangle}{2016} - \frac{\langle 1221222 \rangle}{5040} + \frac{\langle 1222222 \rangle}{30240} \right) \right], \text{True} \right\}$$