From talk by Eddy Godelle.

$$vB_n = \langle \underset{\underset{braiding}{\downarrow}}{\sigma_i}, \underset{\overset{perm}{\nwarrow}}{\tau_i} \rangle / rels$$

$$vB_n \xrightarrow{\;vS\;} S_n \quad by \quad \sigma_i \longmapsto 1, \quad \tau_i \longmapsto (i, i+1)$$
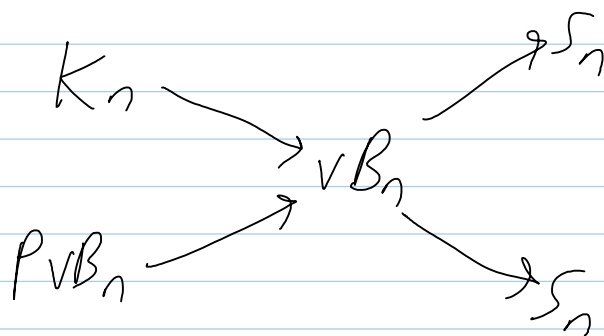
"vertical smoothing map"

$$K_n := ker(vS)$$

generated by v-conjugates of $\times$?

So $\sigma_{ij} \longmapsto (ij)$

$$\sigma_{12} \sigma_{13} \sigma_{23} \longmapsto (12)(13)(23) = (13)$$
$$(23)(13)(12) = (13)$$

Q. what is proj K_n?

Probably 0, as the relations have non-trivial Abelianization.