Sakie Suzuki on the Universal sl2 Invariant of Brunnian Bottom Tangles

June-11-12
3:51 AM

1. Bottom Tangles, Brunnian bottom tangles
2. motivation
3. Univ. sl_2 invariant.
4. Result

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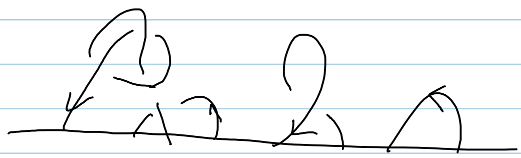
1. Bottom tangles



1. only arcs (no cycles)
2. 2 adjacent endpoints per arc, oriented right to left.

Closure: $\left\{\begin{matrix}n\text{-comp.}\\ \text{bottom}\\ \text{tangles}\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}n\text{-component}\\ \text{links}\end{matrix}\right\}$

$\qquad\qquad\quad || \qquad\qquad\qquad\qquad ||$

$\qquad\qquad\quad BT_n \qquad\qquad\qquad\quad \mathcal{L}_n$

<u>Def</u> L is "Brunnian" link or bottom tangle if all proper sublinks/subtangles thereof are trivial.

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2. Motivation.

1984: The Jones polynomial.

.... the colored Jones polynomial.

$$BT_n \xrightarrow{cl} \mathcal{L}_n$$

$$\begin{array}{ccc} BT_n & \xrightarrow{\ cl\ } & L_n \\ \Big\downarrow J_T & & \Big\downarrow J_{V_1 \ldots V_n} \\ U_h(sl_2)^{\otimes n} & \xrightarrow[\ Tr_q^{V_1}\otimes\cdots\otimes Tr_q^{V_n}\ ]{} & \mathbb{Z}[q^{\pm 1/4}] \end{array}$$

Interest: The relationship between top. props of link & tangles & alg. props of J_T.

Today: An alg. prop. of J_T of Brunnian tangles.

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3. The univ. sl_2 invariant.

<u>Def</u> $U_\hbar(sl_2) = U_h :=$ $\hbar$-adic completed alg over $\mathbb{Q}[[\hbar]]$ generated by H, E, F, with

$$HE - EH = 2E \qquad\qquad q = e^{\hbar}$$
$$[H,F] = -2F \qquad \text{with} \qquad K = e^{\hbar H/2}$$
$$[E,F] = \frac{K - K^{-1}}{q^{1/2} - q^{-1/2}}$$

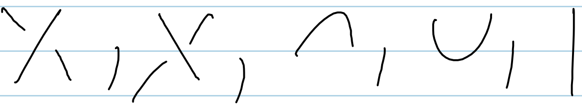
$U_\hbar$ is a Ribbon Hopf Algebra:

$$(U_\hbar, \eta, \mu, \epsilon, \Delta, \underset{\substack{\uparrow \\ U_h^{\otimes 2}}}{R}, \underset{\substack{\uparrow \\ U_h}}{r})$$

Univ. sl_2 invariant: (of $T \in BT_n$)

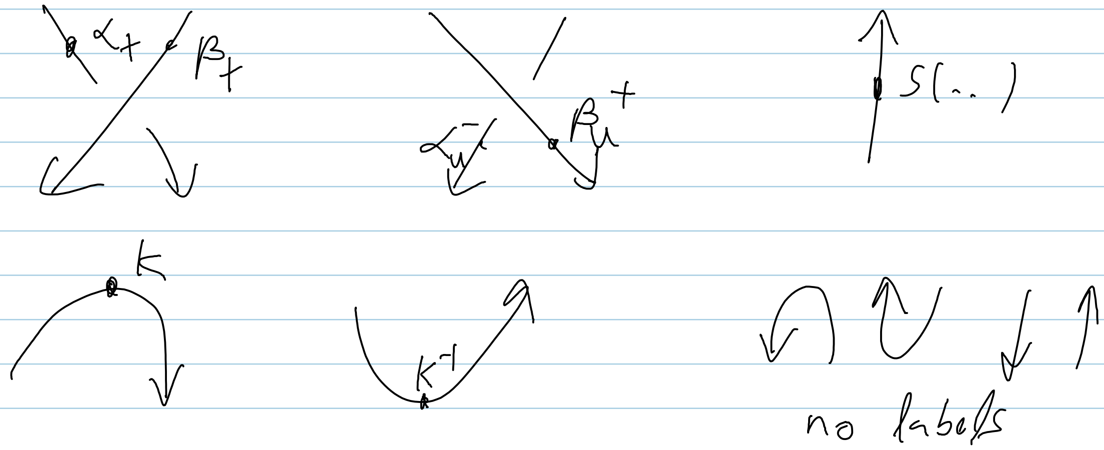
1. Chose a diagram $\tilde{T}$ of T, with

1. Chose a diagram ⊤ or ↑, with parts



2. Put labels.

$$R^{\pm} = \sum \alpha_i^{\pm} \otimes \beta_i^{\pm}$$



3. Read the labels, sum & multiply.

recipe from our high priests.

<u>Notation</u> $[i]_q := \dfrac{q^i - 1}{q - 1}$ $\quad [i_q]! = [i]_q [i-1]_q \ldots [1]_q$

$$\tilde{F}^{(i)} = \frac{F^i K^i}{[i]_q!} \qquad \tilde{E}^{(i)} = \frac{(q^{-1/2} E)^i}{[i]_q!}$$

$$f = (q-1) F K \qquad e = q^{-1/2}(q-1) E$$

$$D = q^{\frac{H \otimes H}{4}} = e^{\frac{H \otimes H}{4} h}$$

$$R = D \cdot \sum_{i \geq 0} q^{\frac{1}{2} i (i-1)} \tilde{F}^{(i)} K^{-i} \otimes e^i$$

$$R^{-1} = D^{-1} \sum_{i \geq 0} (-1)^i \tilde{F}^{(i)} \otimes K^{-i} e^i$$

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4. Results.

Def $\mathbb{Z}[q^{\pm}]$-subalgebras of U_h:

	generators			
$U_{\mathbb{Z}q}$	$K^{\pm 1}$,	$\tilde{F}^{(i)}$,	$\tilde{E}^{(i)}$	$i \geq 0$
$\cup$ U_q	"	"	e	$i \geq 0$
$\cup$ $\bar{U}_q$	"	F	e	$i \geq 0$

superscrip "ev" means $K^{\pm 1} \to K^{\pm 2}$

Prop If $T \in BT_n$ is alg-split (0-framed, 0-linking matrix)

Then

[Habiro] $J_T \in (U_q^{ev})^{\otimes n}$

[Suzuki] Each individual state in the sum is in same.

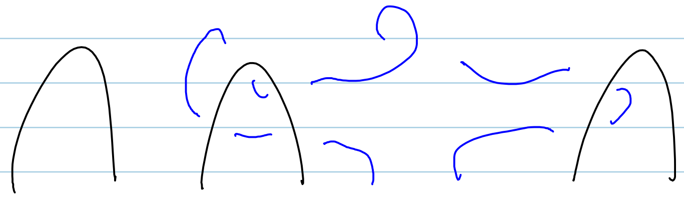
Thm (Suzuki) Let T be an n-comp. Brunnian bottom tangle.

① $\exists$ diagram $\tilde{T}_i$ s.t. for every state s,

$$J_{\tilde{T},s} \in (\bar{U}_q^{ev})^{\otimes i-1} \otimes U_{\mathbb{Z},q}^{ev} \otimes (\bar{U}^{ev})^{\otimes(n-i)} =: B_i^{(n)}$$

2. $J_T \in \bigcap_{i=1}^{n} \{B_i^{(n)}\}^{\wedge}$

Hint for the proof:

$\tilde{T}_i \sim$ 

only the i'th component travels.