Meta-Groups, Meta-Bicrossed-Products, and the Alexander Polynomial, 1

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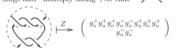
Abstract. The a priori expectation of first year elementary school Alexander Issues.

students who were just introduced to the natural numbers, if they would be ready to verbalize it, must be that soon, perhaps by second grade, they'd master the theory and know all there is to know about those numbers. But they would be wrong, for number that the control of the co

theory remains a thriving subject, well-connected to practically anything there is out there in mathematics. was a bit more sophisticated when I first heard of knot theory My first thought was that it was either trivial or intractable, and most definitely, I wasn't going to learn it is interesting. But it is and I was wrong, for the reader of knot theory is often lead to the most interesting and beautiful structures in topology, geometry

Idea. Given a group G and two "YB" Idea. Given a group G and two "YB" Today I will talk about just one minor example, mostly having pairs $R^{\pm}=(g_{\sigma}^{\pm},g_{\pi}^{\pm})\in G^2$, map them to do with the link to algebra: A straightforward proposal for a group-theoretic invariant of knots fails if one really means groups but works once generalized to meta-groups (to be defined). We will construct one complicated but elementary meta-group as a meta-bicrossed-product (to be defined), and explain how the resulting invariant is a not-vet-understood vet potentially significant gen

invariant is a non-ver-understood yet potentially significant generalization of the Alexander polynomial, while at the same time being a specialization of a somewhat-understood "universal finite This Fails! R2 implies that $g_o^{\pm}g_o^{\mp}=e=g_u^{\pm}g_u^{\mp}$ and then R3 type invariant of w-knots" and of an elusive "universal finite type implies that g_o^{\pm} and g_u^{\pm} commute, so the result is a simple nvariant of v-knots"

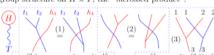


Also has S_x for inversion, e_x for unit insertion, d_x for register dele tion, Δ_{xy}^z for element cloning, ρ_y^z for renamings, and $(D_1, D_2) = D_1 \cup D_2$ for merging, and many obvious composition axioms rela ing those. $P = \{x: g_1, y: g_2\} \Rightarrow P = \{d_yP\} \cup \{d_xP\}$

A Meta-Group. Is a similar "computer", only its internal structure is unknown to us. Namely it is a collection of sets $\{G_{\gamma}\}$ indexed by all finite sets γ , and a collection of operations m_z^{xy} , S_x , e_x , d_x , Δ_{xy}^z (sometimes), ρ_y^x , and \cup , satisfying the exact same linear properties.

Example 1. The non-meta example, $G_{\gamma} := G^{\gamma}$.

Bicrossed Products. If G = HT is a group presented as a product of two of its subgroups, with $H \cap T = \{e\}$, then also G = TH and G is determined by H, T, and the "swap" map $sw^{th}: (t,h) \mapsto (h',t')$ defined by th = h't'. The map sw satisfies (1) and (2) below; conversely, if $sw: T \times H \to H \times T$ satisfies (1) and (2) (+ lesser conditions), then (3) defines a group structure on $H \times T$, the "bicrossed product".



Jana Archibald.

Broken surface

2D Symbol

Dim. reduc.

Virtual crossing Movie 0 "divide and conquer R3 / R2

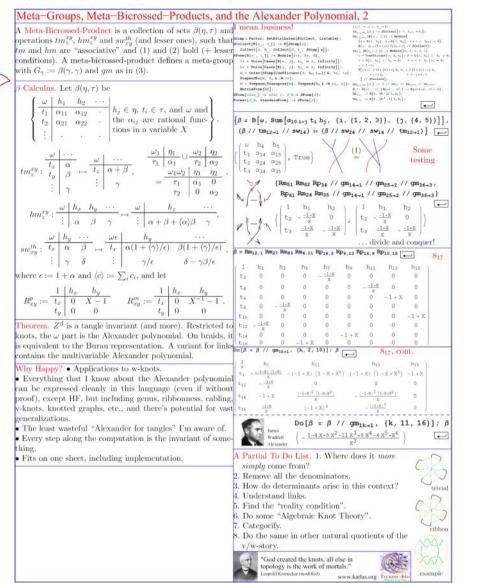
A Standard Alexander Formula. Label the arcs 1 through (n+1) = 1, make an $n \times n$ matrix as below, delete one row unn operations, and "block diagonal" merges. A Standard Alexander Formula. and one column, and compute the determinant:

[[1::7, 1::7]] // Det -

Note: A moto-group

plus a YB pair give a

knot/tangle invariant.



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N	leta-Groups, Meta-Bicrossed-Products, and	the Alexander Polynomial, 3	
	here does it come from? The accidental ¹ answer is that it a symbolic calculus for a natural reduction ⁴ of the unique		
ho	momorphic expansion ² of w-tangles ³ .		
	"Accidental" for it's only how I came about it. There ought to be a better answer.		
2.	A "homomorphic expansion", aka as a homomorphic universal finite type invariant, is a completely canonical con-		
		<< KnotTheory`	
	"v-Tangles" are the meta-group generated by crossings modulo Reidemeister moves. "w-Tangles" are a natural		
	hans identical to a certain class of 1D/2D knots in 4D	Read more at http://katlas.org/wiki/KnotTheory. $-\frac{1-4x\cdot8x^2-11x^3\cdot8x^4-4x^5\cdot x^6}{x^3}$	
	To "only what is visible by the 2D Lie algebra". certain generalization will arise by not reducing as in 4. A		
va: for	st generalization may arise when homomorphic expansions		
Et	ingof-Kazhdan quantization of Lie bialgebras.	The key trick: $\frac{\omega \mid h_j}{t_i \mid \alpha_{ij}} \longleftrightarrow B(\omega, \Lambda = \sum_{i,j} \alpha_{ij} t_i h_j).$	