Jean-Baptiste Meilhan on Milnor Invariants and the HOMFLYPT Polynomial

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(joint w/ Akira Yasahara)

1. Milnor invariants

L = oriented ordered n-comp. link in S^3

$\pi = \pi_1(S^3 / L)$

$$\pi = \Gamma_1 \pi \supset \Gamma_2 \pi \supset \dots \supset \underset{\displaystyle [\pi, \Gamma_k \pi]}{\overset{\shortparallel}{\Gamma_{k+1}\pi}}$$

For each $k \geq 2$, $\pi / \Gamma_k \pi$ is generated by the meridians $m_1 \dots m_n$.

In particular, the longtitude l_j is a word w_j in $\pi/\Gamma_k \pi$ - - --

Use it & magnus expansion to get integers:

$$E(w_j) = 1 + \sum_{i_1 \dots i_k} \mu_L(i_1 \dots i_k j) X_{i_1} \dots X_{i_k}$$

magnus expansion

$\Delta(I) := \gcd\{\mu_L(J) : J$ is a subcyclic word of $I\}$

<u>Claim</u> $\bar{\mu}_L(I) := \mu_L(I) \bmod \Delta$ is a link invariant.

Relation to the Alexander polynomial:

L: n comp. link; assume $\bar{\mu}_L(I) = 0$ $\forall\, |I| \le k$.

Then: $\nabla_L(z)$ is divisible by $z^{(n-1)k}$

and $$\operatorname{coeff}\left(z^{(n-1)k}\right) = \det(a_{ij})$$

where
$$a_{ij} = \begin{cases} \sum_{i_1 \ldots i_{k-1}} \bar{\mu}_L(i\, i_1 \ldots i_{k-1}\, j) & k \geq 2 \\ \text{determined by } lk & k = 1 \end{cases}$$

[$k=2, n=3$ — Cochran]
[J. Levine 99]
[Hoste 85]

HOMFLYPT polynomial:

$$t\,P(\nwarrow\!\!\!\!\nearrow) - t^{-1}P(\nwarrow\!\!\!\!\nearrow) = z\,P(\uparrow\uparrow)$$

$$P(\bigcirc) = 1$$