Emmanuel Wagner on the Jaeger Expansion of Kauffman Polynomial

j/ w/ Pedro Vaz

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5:07 AM

Schur-Weyl Duality: $sl_2 \circlearrowright V^{\otimes d} \circlearrowleft S_d$

$$so(2n) \circlearrowright V^{\otimes d} \circlearrowleft Brauer_d$$

Quantum:

$$U_q(sl_n) \circlearrowright V^{\otimes d} \circlearrowleft Hecke_d(q)$$

$$U_q(so(2n)) \circlearrowright V^{\otimes d} \circlearrowleft BMW_d(q)$$

Topology.

HOMFLYPT

$$P(\nwarrow\!\!\!\!\nearrow) - P(\nearrow\!\!\!\!\nwarrow) = (q-q^{-1})P(\uparrow\uparrow)$$

$$P(\circlearrowright) = aP(\uparrow) \qquad P(\circlearrowleft) = a^{-1}P(\uparrow)$$

$$P(\bigcirc) = \frac{a-a^{-1}}{q-q^{-1}} = [a] =: [a,0]$$

2-variable Kauffman:

$$F(\times) - F(\times) = (q+q^{-1})(F()() - F(\asymp))$$

$$F(\circlearrowright) = \lambda^{-1}F(|) \qquad F(\circlearrowleft) = \lambda F(|)$$

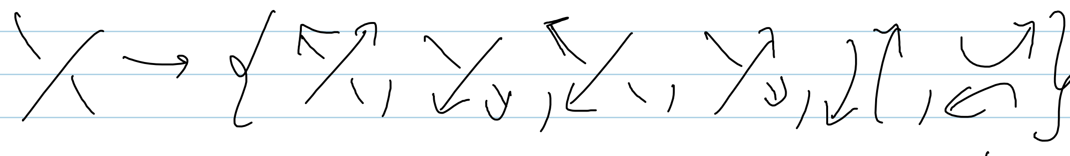
$$F(\bigcirc) = \frac{\lambda-\lambda^{-1}}{q-q^{-1}} + 1$$

$F \in \mathbb{C}(\lambda^{\pm 1}, q^{\pm 1})$ set $\lambda = a^2 q^{-1}$

$$F(\bigcirc) = \frac{a^2 q^{-1} - a^{-2} q}{q - q^{-1}} + 1 = [a^2; -1] + 1$$

<u>Jaeger's Expansion</u> [in Kauffman's Books & physics book]

"$F(D) = \sum_{\vec{D} \subset D} a(\vec{D})\, P(\vec{D})$" ... needs detail.



w $\quad 1 \quad 1 \quad 1 \quad 1 \quad q - q^{-1} \quad q^{-1} - q$

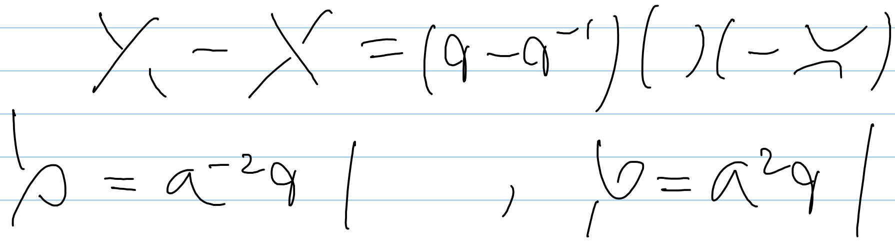
Let Res(D) be the set of all orientation-consistent resolutions of D w/ local resolutions as above.

weight per xing

$$a(\vec{D}) = \prod w_i \,(a^{-1}q)^{\mathrm{rot}(\vec{D} / \cdots \to \cdots)}$$

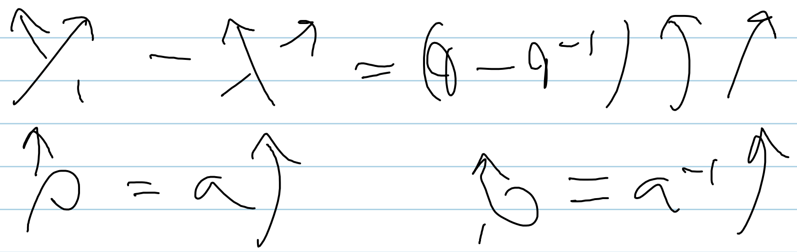
where $\mathrm{rot}(\circlearrowleft) = 1$, $\mathrm{rot}(\circlearrowright) = -1$

$BMW_n(a, q)$ = algebra generated by isotopy classes of framed unoriented (n,n)-tangles modulo



$\dim = (2n-1)!!$

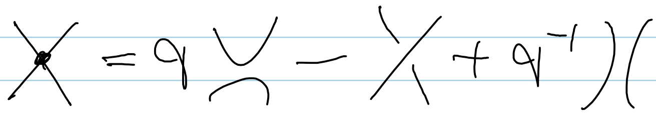
<u>Def</u> $\mathrm{Skein}_n(a,q)$ is the alg. generated by framed oriented (n,n)-tangles mod the HOMFLY relations:



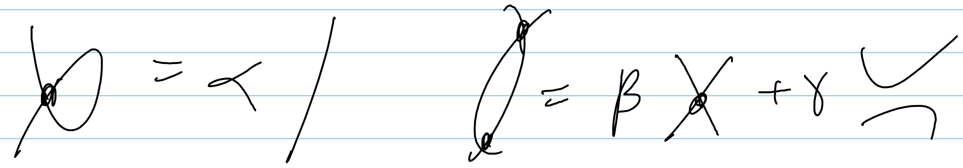
<u>Thm</u> $\exists$ an injective alg. morphism

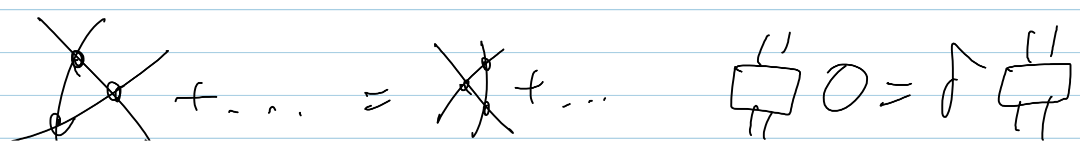
$$\Pi : BMW_n(a,q) \longrightarrow \mathrm{Skein}_n(a,q)$$

[Using Jaeger's expansion]



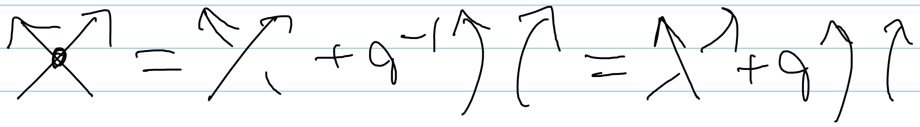
change of basis



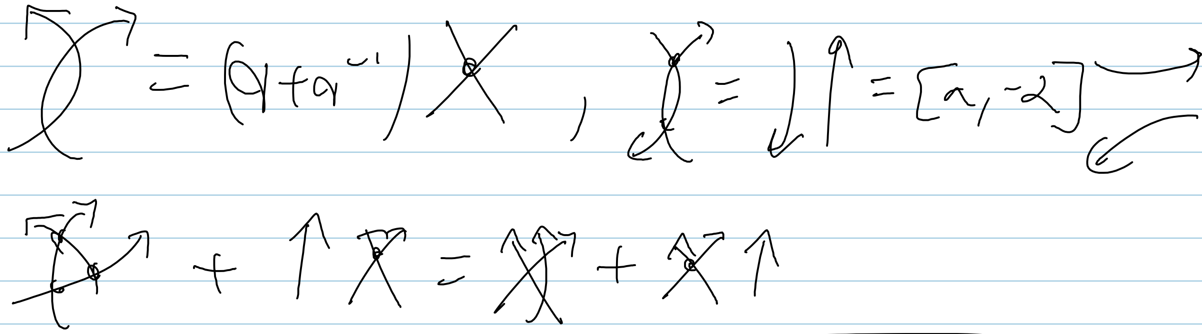


For some coefficients. [for BMW]

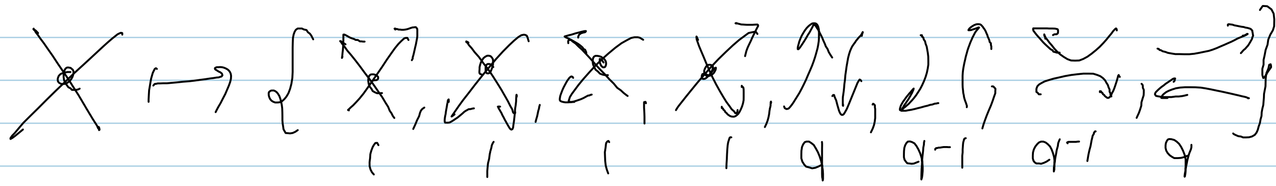
For $\mathrm{Skein}_n(a,q)$:



get



Jaeger's expansion makes sense also after the change of variables:



all weights are now positive!

Categorified World: See video...

...

Want $\mathcal{C}$ to have the Krull-Schmidt property

"unique decomposition into indecomposibles"

<u>Prop</u> $\exists$ a categorification of $BMW_m(q^n, q)$

(yet no proof of Krull-Schmidt)