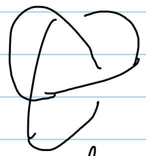
Bellingeri - Groups of Virtual and Welded Links

June-06-12
5:10 AM

Following http://arxiv.org/abs/1204.3205

$\pi_1\Big($  $\Big) \simeq \mathbb{Z}$

(virtual trefoil)

Q. Is there a bijection between w-links and "Wirtinger groups"?

Kamada: A characterisation of w-braids that have the same w-link closure (i.e., Kamada has a Markov thm for w-braids.)

From paper:

Theorem 2. [2] *There is a representation ψ of VB_n in $Aut(F_{n+1})$, $F_{n+1} = \langle x_1, x_2, \ldots, x_n, y\rangle$ which is defined by the following actions on the generators of VB_n:*

$$\psi(\sigma_i) : \begin{cases} x_i \longmapsto x_i\, x_{i+1}\, x_i^{-1}, \\ x_{i+1} \longmapsto x_i, \\ x_l \longmapsto x_l, \ l \neq i, i+1; \\ y \longmapsto y, \end{cases} \qquad \psi(\rho_i) : \begin{cases} x_i \longmapsto y\, x_{i+1}\, y^{-1}, \\ x_{i+1} \longmapsto y^{-1}\, x_i\, y, \\ x_l \longmapsto x_l, \ l \neq i, i+1, \\ y \longmapsto y, \end{cases}$$

for all $i = 1, 2, \ldots, n-1$.

???

$$\begin{tikzcd} F_{n+1} \arrow[r, "\text{``forget } y\text{''}"] \arrow[d, "vB_n"'] & F_n \arrow[d, "wB_n"] \\ F_{n+1} \arrow[r] & F_n \end{tikzcd}$$

Commutes

So if $vB_n \curvearrowright F_{n+1}$ is faithful, then there is a faithful representation of vB_n in wB_{n+1} !

Also by "markov", get an "extended fundamental group" invariant of v-braids. $G_v(vK)$

Does not detect the kishino knot.

[Though adding a "peripheral system", kishino is detected].

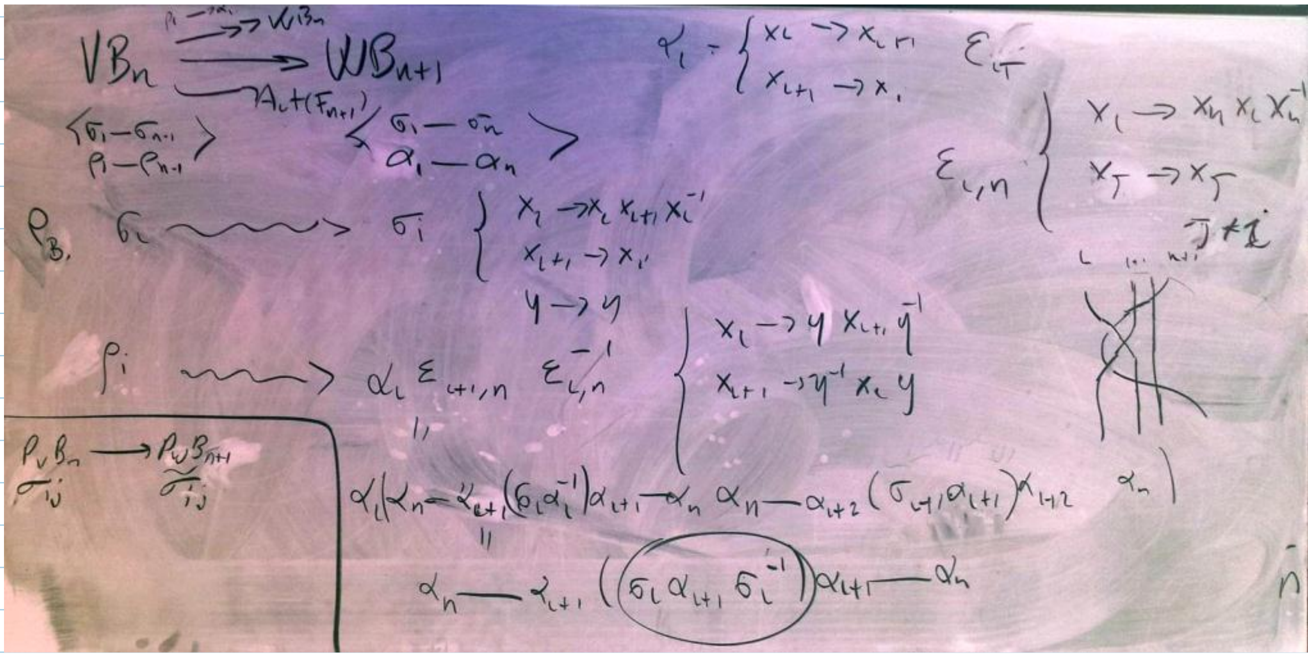
<u>claim</u> $G_v(vK)/\langle\langle y \rangle\rangle \cong \pi_1(vK)$

Wada representations: [of uB_n]

$$x_i \sigma_i = u(x_i, x_{i+1})$$
$$x_{i+1} \sigma_i = v(x_i, x_{i+1}) \qquad u, v \in F_2$$
$$x_j \sigma_i = x_j \quad \text{otherwise.}$$

<u>Prop</u> (Wada) There are at least 7 type of Wada reps.

Ito (2011) There are exactly 7 such reps, up to involusions $x_i \to x_i^{-1}$ & $\sigma_i \to \sigma_i^{-1}$ of F_n or/and B_n.


VB_n → WB_{n+1}
Aut(F_{n+1})
⟨σ_1 — σ_{n-1}, ρ_1 — ρ_{n-1}⟩
⟨σ_1 — σ_n, α_1 — α_n⟩
α_i = { x_i → x_{i+1}, x_{i+1} → x_i }
ε_{i,n} { x_i → x_n x_i x_n^{-1}, x_j → x_j, j ≠ i }
ρ_B, σ_i ⇝ σ_i { x_i → x_i x_{i+1} x_i^{-1}, x_{i+1} → x_i, y → y }
ρ_i ⇝ α_i ε_{i+1,n} ε_{i,n}^{-1}
{ x_i → y x_{i+1} y^{-1}, x_{i+1} → y^{-1} x_i y }
PvB_n → PwB_{n+1}
α_i α_n — α_{i+1} (σ_i α_i^{-1}) α_{i+1} — α_n α_n — α_{i+2} (σ_{i+1} α_{i+1}) α_{i+2} α_n
α_n — α_{i+1} ((σ_i α_{i+1} σ_i^{-1}) α_{i+1} — α_n