Quantum gauge fields and flat connections in 2-dimensional BF theory

Anton Alekseev, Nevena Ilieva
(Submitted on 30 Nov 2010)

The 2-dimensional BF theory is both a gauge theory and a topological Poisson σ-model corresponding to a linear Poisson bracket. In \cite{To1}, Torossian discovered a connection which governs correlation functions of the BF theory with sources for the B-field. This connection is flat, and it is a close relative of the KZ connection in the WZW model. In this paper, we show that flatness of the Torossian connection follows from (properly regularized) quantum equations of motion of the BF theory.

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Configuration space integral for long n-knots, the Alexander polynomial and knot space cohomology

Tadayuki Watanabe
(Submitted on 27 Sep 2006 (v1), last revised 2 Oct 2006 (this version, v2))

There is a higher dimensional analogue of the perturbative Chern-Simons theory in the sense that a similar perturbative series as in 3-dimension, which is computed via configuration space integral, yields an invariant of higher dimensional knots (Bott-Cattaneo-Rossi invariant), which is constructed by Bott for degree 2 and by Cattaneo-Rossi for higher degrees. However, its feature is yet unknown. In this paper we restrict the study to long ribbon n-knots and characterize the Bott-Cattaneo-Rossi invariant as a finite type invariant of long ribbon n-knots in [HKS]. As a consequence, we obtain a non-trivial description of the Bott-Cattaneo-Rossi invariant in terms of the Alexander polynomial.
The results for higher codimension knots are also given. In those cases similar differential forms to define Bott-Cattaneo-Rossi invariant yields infinitely many cohomology classes of Emb(R^n, R^m) if m,n>= 3 odd and m>n+2. We observe that half of these classes are non-trivial, along a line similar to Cattaneo-CottaRamusino-Longoni [CCL].

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Exotic Statistics for Strings in 4d BF Theory

John C. Baez, Derek K. Wise, Alissa S. Crans
(Submitted on 21 Mar 2006 (v1), last revised 9 May 2006 (this version, v2))

After a review of exotic statistics for point particles in 3d BF theory, and especially 3d quantum gravity, we show that string-like defects in 4d BF theory obey exotic statistics governed by the 'loop braid group'. This group has a set of generators that switch two strings just as one would normally switch point particles, but also a set of generators that switch two strings by passing one through the other. The first set generates a copy of the symmetric group, while the second generates a copy of the braid group. Thanks to recent work of Xiao-Song Lin, we can give a presentation of the whole loop braid group, which turns out to be isomorphic to the 'braid permutation group' of Fenn, Rimanyi and Rourke. In the context 4d BF theory this group naturally acts on the moduli space of flat G-bundles on the complement of a collection of unlinked unknotted circles in R^3. When G is unimodular, this gives a unitary representation of the loop braid group. We also discuss 'quandle field theory', in which the gauge group G is replaced by a quandle.

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BF system - encyclopedic entry

Boguslaw Broda
(Submitted on 3 Feb 2005)

The notion of the BF (topological) gauge field theory is defined.

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On Chern-Simons theory with an inhomogeneous gauge group and BF theory knot invariants

Gad Naot
(Submitted on 22 Oct 2003 (v1), last revised 7 Dec 2005 (this version, v3))

We study the Chern-Simons topological quantum field theory with an inhomogeneous gauge group, a non-semi-simple group obtained from a semi-simple one by taking its semi-direct product with its Lie algebra. We find that the standard knot observables (i.e. traces of holonomies along knots) essentially vanish, but yet, the non-semi-simplicity of our gauge group allows us to consider a class of un-orthodox observables which breaks gauge invariance at one point and which lead to a non-trivial theory on long knots in $\mathbb{R}^3$. We have two main morals : 1. In the non-semi-simple case, there is more to observe in Chern-Simons theory! There might be other interesting non semi-simple gauge groups to study in this context beyond our example. 2. In our case of an inhomogeneous gauge group, we find that Chern-Simons theory with the un-orthodox observable is actually the same as 3D BF theory with the Cattaneo-Cotta-Ramusino-Martellini knot observable. This leads to a simplification of their results and enables us to generalize and solve a problem they posed regarding the relation between BF theory and the Alexander-Conway polynomial. Our result is that the most general knot invariant coming from pure BF topological quantum field theory is in the algebra generated by the coefficients of the Alexander-Conway polynomial.

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Wilson surfaces and higher dimensional knot invariants

Alberto S. Cattaneo, Carlo A. Rossi

(Submitted on 20 Oct 2002)

An observable for nonabelian, higher-dimensional forms is introduced, its properties are discussed and its expectation value in BF theory is described. This is shown to produce potential and genuine invariants of higher-dimensional knots.

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Higher-dimensional BF theories in the Batalin-Vilkovisky formalism: The BV action and generalized Wilson loops

Alberto S. Cattaneo, Carlo A. Rossi

(Submitted on 17 Oct 2000 (v1), last revised 11 Apr 2001 (this version, v2))

This paper analyzes in details the Batalin-Vilkovisky quantization procedure for BF theories on n-dimensional manifolds and describes a suitable superformalism to deal with the master equation and the search of observables. In particular, generalized Wilson loops for BF theories with additional polynomial B-interactions are discussed in any dimensions. The paper also contains the explicit proofs to the Theorems stated in math.QA/0003073.

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Loop observables for BF theories in any dimension and the cohomology of knots

Alberto S. Cattaneo, Paolo Cotta-Ramusino, Carlo A. Rossi

(Submitted on 13 Mar 2000)

A generalization of Wilson loop observables for BF theories in any dimension is introduced in the Batalin-Vilkovisky framework. The expectation values of these observables are cohomology classes of the space of imbeddings of a circle. One of the resulting theories discussed in the paper has only trivalent interactions and, irrespective of the actual dimension, looks like a 3-dimensional Chern-Simons theory.

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Loop and Path Spaces and Four-Dimensional BF Theories: Connections, Holonomies and Observables

A. S. Cattaneo, P. Cotta-Ramusino, M. Rinaldi

(Submitted on 17 Mar 1998)

We study the differential geometry of principal G-bundles whose base space is the space of free paths (loops) on a manifold M. In particular we consider connections defined in terms of pairs (A,B), where A is a connection for a fixed principal bundle P(M,G) and B is a 2-form on M. The relevant curvatures, parallel transports and holonomies are computed and their expressions in local coordinates are exhibited. When the 2-form B is given by the curvature of A, then the so-called non-abelian Stokes formula follows.

For a generic 2-form B, we distinguish the cases when the parallel transport depends on the whole path of paths and when it depends only on the spanned surface. In particular we discuss generalizations of the non-abelian Stokes formula. We study also the invariance properties of the (trace of the) holonomy under suitable transformation groups acting on the pairs (A,B). In this way we are able to define observables for both topological and non-topological quantum field theories of the BF type. In the non topological case, the surface terms may be relevant for the understanding of the quark-confinement problem. In the topological case the (perturbative) four-dimensional quantum BF-theory is expected to yield invariants of imbedded (or immersed) surfaces in a 4-manifold M.

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Abelian BF Theories and Knot Invariants

Alberto S. Cattaneo (Harvard University)

(Submitted on 25 Sep 1996 (v1), last revised 4 Apr 1997 (this version, v2))

In the context of the Batalin-Vilkovisky formalism, a new observable for the Abelian BF theory is proposed whose vacuum expectation value is related to the Alexander-Conway polynomial. The three-dimensional case is analyzed explicitly, and it is proved to be anomaly free. Moreover, at the second order in perturbation theory, a new formula for the second coefficient of the Alexander-Conway polynomial is obtained. An account on the higher-dimensional generalizations is also given.

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Cabled Wilson Loops in BF Theories

Alberto S. Cattaneo (Harvard University)

(Submitted on 6 Feb 1996)

A generating function for cabled Wilson loops in three-dimensional BF theories is defined, and a careful study of its behavior for vanishing cosmological constant is performed. This allows an exhaustive description of the unframed knot invariants coming from the pure BF theory based on SU(2), and in particular, it proves a conjecture relating them to the Alexander-Conway polynomial.

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Topological BF Theories in 3 and 4 Dimensions

Aberto S. Cattaneo, Paolo Cotta-Ramusino, Juerg Froehlich, Maurizio Martellini

(Submitted on 4 May 1995 (v1), last revised 5 May 1995 (this version, v2))

In this paper we discuss topological BF theories in 3 and 4 dimensions. Observables are associated to ordinary knots and links (in 3 dimensions) and to 2-knots (in 4 dimensions). The vacuum expectation values of such observables give a wide range of invariants. Here we consider mainly the 3-dimensional case, where these invariants include Alexander polynomials, HOMFLY polynomials and Kontsevich integrals.

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BF Theories and 2-knots

P. Cotta-Ramusino, M. Martellini

(Submitted on 16 Jul 1994)

We discuss the relations between (topological) quantum field theories in 4 dimensions and the theory of 2-knots (embedded 2-spheres in a 4-manifold). The so-called BF theories allow the construction of quantum operators whose trace can be considered as the higher-dimensional generalization of Wilson lines for knots in 3-dimensions. First-order perturbative calculations lead to higher dimensional linking numbers, and it is possible to establish a heuristic relation between BF theories and Alexander invariants. Functional integration-by-parts techniques allow the recovery of an infinitesimal version of the Zamolodchikov tetrahedron equation, in the form considered by Carter and Saito.

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Three-dimensional BF Theories and the Alexander-Conway Invariant of Knots

A. S. Cattaneo, P. Cotta-Ramusino, M. Martellini

(Submitted on 13 Jul 1994)

We study 3-dimensional BF theories and define observables related to knots and links. The quantum expectation values of these observables give the coefficients of the Alexander-Conway polynomial.

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