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In[1]:= SetDirectory["C:/drorbn/AcademicPensieve/2012-06/Regina_Talk/"]
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Out[1]= C:\drorbn\AcademicPensieve\2012-06\Regina_Talk
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StandardAlexander

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In[2]:= 
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & X-1 & 0 & -X \\ -1 & X & 0 & 0 & 0 & 0 & 1-X & 0 \\ 0 & -1 & X & 0 & 1-X & 0 & 0 & 0 \\ X-1 & 0 & -X & 1 & 0 & 0 & 0 & 0 \\ 0 & 1-X & 0 & -1 & X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -X & 1 & 0 & X-1 \\ 0 & 0 & 1-X & 0 & 0 & -1 & X & 0 \\ 0 & 0 & 0 & X-1 & 0 & 0 & -X & 1 \end{pmatrix} \quad [[1 ;; 7, 1 ;; 7]] // \text{Det}$$

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StandardAlexander

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Out[2]= -1 + 4 X - 8 X^2 + 11 X^3 - 8 X^4 + 4 X^5 - X^6
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Initialization

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In[3]:=  $\beta\text{Simp} = \text{Factor}; \text{SetAttributes}[\beta\text{Collect}, \text{Listable}];$ 
 $\beta\text{Collect}[B[\omega_, \Lambda_]] := B[\beta\text{Simp}[\omega],$ 
 $\text{Collect}[\Lambda, h_, \text{Collect}[\#, t_, \beta\text{Simp}] \&]]];$ 
 $\beta\text{Form}[B[\omega_, \Lambda_]] := \text{Module}[\{ts, hs, M\},$ 
 $ts = \text{Union}[\text{Cases}[B[\omega, \Lambda], t_{s_} \rightarrow s, \text{Infinity}]]];$ 
 $hs = \text{Union}[\text{Cases}[B[\omega, \Lambda], h_{s_} \rightarrow s, \text{Infinity}]]];$ 
 $M = \text{Outer}[\beta\text{Simp}[\text{Coefficient}[\Lambda, h_{\#1} t_{\#2}]] \&, hs, ts];$ 
 $\text{PrependTo}[M, t_{\#} \& /@ ts];$ 
 $M = \text{Prepend}[\text{Transpose}[M], \text{Prepend}[h_{\#} \& /@ hs, \omega]]];$ 
 $\text{MatrixForm}[M]]];$ 
 $\beta\text{Form}[\text{else}_] := \text{else} /. \beta\_B \rightarrow \beta\text{Form}[\beta];$ 
 $\text{Format}[\beta\_B, \text{StandardForm}] := \beta\text{Form}[\beta];$ 
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Program

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In[8]:=  $\langle \mu\_ \rangle := \mu /. t\_ \rightarrow 1;$ 
 $\text{tm}_{x\_y\_z\_}[\beta\_] := \beta\text{Collect}[\beta /. t_{x|y} \rightarrow t_z];$ 
 $\text{hm}_{x\_y\_z\_}[B[\omega_, \Lambda_]] := \text{Module}[\{$ 
 $\alpha = D[\Lambda, h_x], \beta = D[\Lambda, h_y], \gamma = \Lambda /. h_{x|y} \rightarrow 0\},$ 
 $B[\omega, (\alpha + (1 + \langle \alpha \rangle) \beta) h_z + \gamma] // \beta\text{Collect}];$ 
 $\text{sw}_{x\_y\_}[B[\omega_, \Lambda_]] := \text{Module}[\{\alpha, \beta, \gamma, \delta, \epsilon\},$ 
 $\alpha = \text{Coefficient}[\Lambda, h_y t_x]; \beta = D[\Lambda, t_x] /. h_y \rightarrow 0;$ 
 $\gamma = D[\Lambda, h_y] /. t_x \rightarrow 0; \delta = \Lambda /. h_y | t_x \rightarrow 0;$ 
 $\epsilon = 1 + \alpha;$ 
 $B[\omega * \epsilon, \alpha (1 + \langle \gamma \rangle / \epsilon) h_y t_x + \beta (1 + \langle \gamma \rangle / \epsilon) t_x$ 
 $+ \gamma / \epsilon h_y + \delta - \gamma * \beta / \epsilon$ 
 $] // \beta\text{Collect}];$ 
 $\text{gm}_{x\_y\_z\_}[\beta\_] := \beta // \text{sw}_{xy} // \text{hm}_{xy \rightarrow z} // \text{tm}_{xy \rightarrow z};$ 
 $B /: B[\omega1_, \Lambda1_] B[\omega2_, \Lambda2_] := B[\omega1 * \omega2, \Lambda1 + \Lambda2];$ 
 $\text{RP}_{x\_y\_} := B[1, (X-1) t_x h_y];$ 
 $\text{Rm}_{x\_y\_} := B[1, (X^{-1} - 1) t_x h_y];$ 
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$$\text{In[16]:= } \left\{ \beta = \mathbf{B} \left[\omega, \text{Sum} \left[\alpha_{10 \ i+j} \mathbf{t}_i \mathbf{h}_j, \{i, \{1, 2, 3\}\}, \{j, \{4, 5\}\} \right] \right], \right. \\ \left. (\beta // \mathbf{tm}_{12 \rightarrow 1} // \mathbf{sw}_{14}) == (\beta // \mathbf{sw}_{24} // \mathbf{sw}_{14} // \mathbf{tm}_{12 \rightarrow 1}) \right\}$$

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$$\text{Out[16]= } \left\{ \begin{pmatrix} \omega & h_4 & h_5 \\ t_1 & \alpha_{14} & \alpha_{15} \\ t_2 & \alpha_{24} & \alpha_{25} \\ t_3 & \alpha_{34} & \alpha_{35} \end{pmatrix}, \text{ True} \right\}$$

R3

$$\text{In[17]:= } \left\{ \mathbf{Rm}_{51} \mathbf{Rm}_{62} \mathbf{Rp}_{34} // \mathbf{gm}_{14 \rightarrow 1} // \mathbf{gm}_{25 \rightarrow 2} // \mathbf{gm}_{36 \rightarrow 3}, \right. \\ \left. \mathbf{Rp}_{61} \mathbf{Rm}_{24} \mathbf{Rm}_{35} // \mathbf{gm}_{14 \rightarrow 1} // \mathbf{gm}_{25 \rightarrow 2} // \mathbf{gm}_{36 \rightarrow 3} \right\}$$

R3

$$\text{Out[17]= } \left\{ \begin{pmatrix} 1 & h_1 & h_2 \\ t_2 & -\frac{-1+X}{X} & 0 \\ t_3 & -\frac{-1+X}{X} & -\frac{-1+X}{X} \end{pmatrix}, \begin{pmatrix} 1 & h_1 & h_2 \\ t_2 & -\frac{-1+X}{X} & 0 \\ t_3 & -\frac{-1+X}{X} & -\frac{-1+X}{X} \end{pmatrix} \right\}$$

8_17-1

$$\text{In[18]:= } \beta = \mathbf{Rm}_{12,1} \mathbf{Rm}_{27} \mathbf{Rm}_{83} \mathbf{Rm}_{4,11} \mathbf{Rp}_{16,5} \mathbf{Rp}_{6,13} \mathbf{Rp}_{14,9} \mathbf{Rp}_{10,15}$$

8_17-1

$$\text{Out[18]= } \begin{pmatrix} 1 & h_1 & h_3 & h_5 & h_7 & h_9 & h_{11} & h_{13} & h_{15} \\ t_2 & 0 & 0 & 0 & -\frac{-1+X}{X} & 0 & 0 & 0 & 0 \\ t_4 & 0 & 0 & 0 & 0 & 0 & -\frac{-1+X}{X} & 0 & 0 \\ t_6 & 0 & 0 & 0 & 0 & 0 & 0 & -1+X & 0 \\ t_8 & 0 & -\frac{-1+X}{X} & 0 & 0 & 0 & 0 & 0 & 0 \\ t_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+X \\ t_{12} & -\frac{-1+X}{X} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t_{14} & 0 & 0 & 0 & 0 & -1+X & 0 & 0 & 0 \\ t_{16} & 0 & 0 & -1+X & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

8_17-2

$$\text{In[19]:= } \mathbf{Do}[\beta = \beta // \mathbf{gm}_{1k \rightarrow 1}, \{k, 2, 10\}]; \beta$$

8_17-2

$$\text{Out[19]= } \begin{pmatrix} \frac{1}{X} & h_1 & h_{11} & h_{13} & h_{15} \\ t_1 & -\frac{(-1+X)(1+X)}{X} & -(-1+X)(1-X+X^2) & (-1+X)(1-X+X^2) & -1+X \\ t_{12} & -\frac{-1+X}{X} & 0 & 0 & 0 \\ t_{14} & -1+X & \frac{(-1+X)^2(1-X+X^2)}{X} & -\frac{(-1+X)^2(1-X+X^2)}{X} & 0 \\ t_{16} & \frac{-1+X}{X} & (-1+X)^2 & -\frac{(-1+X)^3}{X} & 0 \end{pmatrix}$$

8_17-3

$$\text{In[20]:= } \mathbf{Do}[\beta = \beta // \mathbf{gm}_{1k \rightarrow 1}, \{k, 11, 16\}]; \beta$$

8_17-3

$$\text{Out[20]= } \left(-\frac{1-4X+8X^2-11X^3+8X^4-4X^5+X^6}{X^3} \right)$$