Speyer's suggestion

May-16-12
7:55 PM

$$\underbrace{\begin{array}{c|c|c} W_0 & y & - \\ \hline x & \alpha & \beta \\ \hline 1 & \gamma & \delta \\ \hline \cdot & \sigma_y & \sigma \end{array}}_{A_0} \quad \xrightarrow{\text{swap}^{th}_{xy}} \quad \underbrace{\begin{array}{c|c|c} W_1 = W_0+\alpha & y & - \\ \hline x & \sigma_y \alpha & \sigma_y \beta \\ \hline 1 & \gamma & \frac{(W_0+\alpha)\delta - \gamma\beta}{W_0} \\ \hline \cdot & \sigma_y & \sigma \end{array}}_{A_1}$$

$$\frac{(W_0+\alpha)\delta - \gamma\beta}{W_0} = \delta + \frac{\alpha\delta - \gamma\beta}{W_0}$$

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \gamma_0 \end{pmatrix} \begin{pmatrix} \alpha_0' & \beta_0 \end{pmatrix} + W \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix}$$

$$\boxed{\alpha_0 \alpha_0' = \alpha - W\alpha_1}$$

$$\delta + \frac{\alpha\delta - \gamma\beta}{W} = \frac{(W+\alpha)\delta}{W} - \frac{\gamma\beta}{W} =$$

$$\frac{W+\alpha}{W}\left(\gamma_0\beta_0 + W\delta_1\right) - \frac{1}{W}\left(\left(\gamma_0\alpha_0' + W\gamma_1\right)\left(\alpha_0\beta_0 + W\beta_1\right)\right)$$

$$= (W+\alpha)\delta_1 + \frac{1}{W}\left((W+\alpha)\gamma_0\beta_0 - \alpha\gamma_0\beta_0 + W\alpha_1\gamma_0\beta_0 - \alpha_0 W\gamma_1\beta_0 - \alpha_0' W\gamma_0\beta_1 - W^2\gamma_1\beta_1\right)$$

$$= (W+\alpha)\delta_1 + \gamma_0\beta_0 - \alpha_0\gamma_1\beta_0 - \alpha_0'\gamma_0\beta_1 - W\gamma_1\beta_1 + \alpha_1\gamma_0\beta_0$$

$$= (W+\alpha)\delta_1 + (\gamma_0 - \alpha_0\gamma_1)(\beta_0 - \alpha_0'\beta_1) - \alpha_0\alpha_0'\gamma_1\beta_1 - W\gamma_1\beta_1 + \alpha_1\gamma_0\beta_0$$

$$= (W+\alpha)\delta_1 + (\gamma_0 - \alpha_0\gamma_1)(\beta_0 - \alpha_0'\beta_1') - (W+\alpha)\gamma_1\beta_1 + W\alpha_1\gamma_1\beta_1 + \alpha_1\gamma_0\beta_0$$

$$= (W+\alpha)(\delta_1 - \gamma_1\beta_1) + (\gamma_0 - \alpha_0\gamma_1)(\beta_0 - \alpha_0'\beta_1') + \alpha_1(\delta - W\delta_1 + W\gamma_1\beta_1)$$

So the new μ must be:

$$\begin{pmatrix} \alpha_0' & \beta_0 - \alpha_0'\beta_1 \end{pmatrix}$$

$$\begin{pmatrix} \\ \gamma_0 - \alpha_0\gamma_1 \end{pmatrix} \qquad + (W+\lambda)\begin{pmatrix} & \\ & \delta_1 - \gamma_1\beta_1 \end{pmatrix}$$

Question. Given a matrix M, how do I detect if it is of the form

$$M = \gamma\beta + W M' \quad ?$$

$$\begin{array}{c|c} W_1 & \\ \hline & \gamma_1 \cdot \beta_1 \end{array} \cdot \begin{array}{c|c} W_2 & \\ \hline & \gamma_2\beta_2 \end{array} = \begin{array}{c|cc} W_1W_2 & & \\ \hline & W_2\gamma_1\beta_1 & 0 \\ & 0 & W_1\gamma_2\beta_2 \end{array}$$

$$\neq \quad \begin{array}{c|cc} W_1W_2 & W_2\beta_1 & \beta_2 \\ \hline \gamma_1 & & \\ W_1\gamma_2 & & \end{array}$$