Polynomiality of beta-better

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9:46 AM

Time for action!

$$\underbrace{\begin{array}{c|c|c} W_0 & y & - \\ \hline x & \alpha & \beta \\ \hline 1 & \gamma & \delta \\ \hline \cdot & \sigma_y & \sigma \end{array}}_{A_0} \xmapsto{\text{swap}^{th}_{xy}} \underbrace{\begin{array}{c|c|c} W_1=W_0+\alpha & y & - \\ \hline x & \sigma_y\alpha & \sigma_y\beta \\ \hline 1 & \gamma & \frac{(W_0+\alpha)\delta-\gamma\beta}{W_0} \\ \hline \cdot & \sigma_y & \sigma \end{array}}_{A_1}$$

$$\frac{(W_0+\alpha)\delta-\gamma\beta}{W_0} = \delta + \frac{\alpha\delta-\gamma\beta}{W_0}$$

Have: Complexes for A_0, W_0, B_0 & homotopy

$$\Lambda^2 A_0 \sim W_0 \otimes B_0$$

Want: same for A_1, W_1, B_1

extra info:

$M|_{T=1} = 0$,

$W|_{T=1} = 1$

So $M = (T-1)\ldots$

$W = 1 + (T-1)\ldots$

$3\cdot 2 = 6$

$1 + 3\cdot 1 = 4$ not rel. prime.

$(T-1)\cdot T$

$1 + (T-1)\cdot 1$

$$\begin{vmatrix} ac & ad \\ bc & bd \end{vmatrix} = acbd - adbc = 0$$

$$\begin{vmatrix} 6\delta_1 & ad \\ 6\delta_2 & bd \end{vmatrix} + \begin{vmatrix} ac & 6\delta_3 \\ bc & 6\delta_4 \end{vmatrix} =$$

$$\frac{\alpha\delta-\gamma\beta}{W} = \epsilon \Rightarrow \gamma\beta = \alpha\delta - W\epsilon$$

Ouch!

$$\frac{(W+\alpha)\delta - \alpha\delta + W\epsilon}{W} = \delta + \epsilon$$

$$\frac{W+\alpha}{W} \Big| \Lambda^2\left((W+\alpha)\delta - \gamma\beta\right)$$

$$\alpha\frac{(W+\alpha)\delta - \gamma\beta}{W} - \gamma\beta = \frac{W+\alpha}{W}(\alpha\delta - \gamma\beta) \quad ✓$$

$$\begin{vmatrix} \gamma_1 & \frac{(W+\alpha)\delta_1 - \beta\gamma_1}{W} \\ \gamma_2 & (W+\alpha)\delta_2 - \beta\gamma_2 \end{vmatrix} = \left(\frac{W+\alpha}{W}\right)(\gamma_1\delta_2 - \gamma_2\delta_1) \quad ✓$$

$$\frac{1}{W^2}\begin{vmatrix}(W+\alpha)\delta_{11}-\gamma_1\beta_1 & (W+\alpha)\delta_{12}-\gamma_1\beta_2\\ (W+\alpha)\delta_{21}-\gamma_2\beta_1 & (W+\alpha)\delta_{22}-\gamma_2\beta_2\end{vmatrix} =$$

Use FOIL

$$= \frac{1}{W^2}\Bigg[(W+\alpha)^2\begin{vmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{vmatrix} - (W+\alpha)\begin{vmatrix}\delta_{11} & \gamma_1\beta_2\\ \delta_{21} & \gamma_2\beta_2\end{vmatrix}$$

$$- (W+\alpha)\begin{vmatrix}\gamma_1\beta_1 & \delta_{12}\\ \gamma_2\beta_1 & \delta_{22}\end{vmatrix} + \begin{vmatrix}\gamma_1\beta_1 & \gamma_1\beta_2\\ \gamma_2\beta_1 & \gamma_2\beta_2\end{vmatrix}\Bigg]$$

F & L work well. $?_0$

$$-OI = \frac{1}{W^2}(W+\alpha)\left[\beta_2\begin{vmatrix}\delta_{11} & \gamma_1\\ \delta_{21} & \gamma_2\end{vmatrix} + \beta_1\begin{vmatrix}\gamma_1 & \delta_{12}\\ \gamma_2 & \delta_{22}\end{vmatrix}\right]$$

$$=$$

$$\begin{pmatrix}\alpha & 1 & 0\\ 1 & \delta_{11} & \delta_{12}\\ 0 & \delta_{21} & \delta_{22}\end{pmatrix} \longrightarrow \begin{pmatrix}\alpha & 1 & 0\\ 1 & \frac{1}{W}[(W+\alpha)\delta_{11}-1] & \frac{W+\alpha}{W}\delta_{12}\\ 0 & \frac{W+\alpha}{W}\delta_{21} & \frac{W+\alpha}{W}\delta_{22}\end{pmatrix}$$

W divides $\delta_{12}, \delta_{22}, \delta_{21}, \alpha\delta_{11}-1$

$$\begin{pmatrix}\alpha & \beta & 0\\ 1 & \delta_{11} & \delta_{12}\\ 0 & \delta_{21} & \delta_{22}\end{pmatrix} \longrightarrow \begin{pmatrix}\alpha & \beta & 0\\ 1 & \frac{1}{W}((W+\alpha)\delta_{11}-\beta) & \frac{W+\alpha}{W}\delta_{12}\\ 0 & \frac{W+\alpha}{W}\delta_{21} & \frac{W+\alpha}{W}\delta_{22}\end{pmatrix}$$

$W \mid \delta_{21}, \delta_{22}, \beta\delta_{12}, \alpha\delta_{11}-\beta$

$$\begin{pmatrix} \alpha & \beta & 0 \\ \gamma & \delta_{11} & \delta_{12} \\ 0 & \delta_{21} & \delta_{22} \end{pmatrix} \longrightarrow \begin{pmatrix} \alpha & \beta & 0 \\ \gamma & \frac{1}{w}(w+\alpha)\delta_{11} - \beta\gamma & \frac{w+\alpha}{w}\delta_{12} \\ 0 & \frac{w+\alpha}{w}\delta_{21} & \frac{w+\alpha}{w}\delta_{22} \end{pmatrix}$$

$$w \mid \gamma\delta_{21},\ \gamma\delta_{22},\ \beta\delta_{12},\ \beta\delta_{22},\ \alpha\delta_{11} - \beta\gamma,\ \alpha\delta_{12},\ \alpha\delta_{22},\ \alpha\delta_{21}$$

$$\begin{vmatrix} \frac{1}{w}(w+\alpha)\delta_{11} - \beta\gamma & \frac{w+\alpha}{w}\delta_{12} \\ \frac{w+\alpha}{w}\delta_{21} & \frac{w+\alpha}{w}\delta_{22} \end{vmatrix} = \frac{w+\alpha}{w^2} \begin{vmatrix} (w+\alpha)\delta_{11} - \beta\gamma & \delta_{12} \\ (w+\alpha)\delta_{21} & \delta_{22} \end{vmatrix}$$

$$= \underbrace{\frac{w+\alpha}{w} \begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{vmatrix}}_{\text{ok}} + \underbrace{\frac{w+\alpha}{w^2} \begin{vmatrix} \alpha\delta_{11} - \beta\gamma & \delta_{12} \\ \alpha\delta_{21} & \delta_{22} \end{vmatrix}}_{\text{look for a counter example here.}}$$

Q: $(w+\alpha)\delta_{12}$ & $(w+\alpha)\delta_{22}$ are divisible by w. Does it follow that $\frac{w+\alpha}{w}\delta_{12}$ & $\frac{w+\alpha}{w}\delta_{22}$ are divisible by $w+\alpha$?

$\underset{w}{5} \mid \underset{w+\alpha}{15} \cdot \frac{1}{3}$ yet $15 \nmid 15 \cdot \frac{1}{3}$

$p \mid qa \Rightarrow q \mid \frac{q}{p} a$?

$6 \mid 3 \cdot 2$ yet $3 \nmid \frac{3}{6} \cdot 2$

$$\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \delta_{11} & \delta_{12} \\ 0 & \delta_{21} & \delta_{22} \end{pmatrix} \longrightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \frac{\epsilon}{w}\delta_{11} & \frac{\epsilon}{w}\delta_{12} \\ 0 & \frac{\epsilon}{w}\delta_{21} & \frac{\epsilon}{w}\delta_{22} \end{pmatrix} \qquad \epsilon = w+\alpha$$

$$\frac{(w+\alpha)^2}{w^2} \begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{vmatrix} =$$

3 3

$$\frac{(w+\alpha)^2}{w^2}\begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{vmatrix} = \qquad \begin{matrix} 3 & 3 \\ 3 & -3 \end{matrix}$$

$$\frac{6+2}{6}\begin{vmatrix} 3 & 3 \\ 3 & -3 \end{vmatrix} = \frac{8}{6}\cdot(-18) =$$

Given $w \mid \epsilon\delta_{ij}$ & $w \mid |\delta_{ij}|$, does $\epsilon \mid \frac{\epsilon^2}{w^2}|\delta_{ij}|$?

$p \mid qa,\ p \mid qb,\ p \mid ab \overset{?}{\Longrightarrow} q \mid \frac{q^2}{p^2}ab$

$6 \mid 3\cdot 2 \quad 6 \mid 3\cdot 6,\ 6 \mid 2\cdot 6 \qquad 3 \mid \frac{9}{36}\cdot 12$ ✓

$p \le q+a \quad p \le q+b,\ p \le a+b \Rightarrow q \overset{?}{\le} 2q-2p+a+b$?

$\Downarrow \qquad\qquad 2p \overset{?}{\le} q+a+b$

$3p \le 2(a+b+q) \Rightarrow p \le \frac{2}{3}(q+a+b)$

$\Rightarrow 2p \le \frac{4}{3}(q+a+b)$

take $a=b=q=1,\ p=2$.

$4 \mid 2\cdot 2,\ 4 \mid 2\cdot 2,\ 4 \mid 2\cdot 2$ yet $2 \nmid \frac{2^2}{4^2}\cdot 2\cdot 2$

$$\begin{array}{c|ccc} T^2 & & & \\ \hline & T & & \\ & & T & \\ & & & T \end{array} \longmapsto \begin{array}{c|ccc} T^2+T & & & \\ \hline & T & & \\ & & \frac{T^2+T}{T^2}\cdot T & \\ & & & \frac{T^2+T}{T^2}\cdot T \end{array}$$

$T^2+T \nmid \left(\frac{T^2+T}{T}\right)^2 = (T+1)^2$ yet mod units this works.

yet again, this example does not satisfy the "blue condition".