```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05"];
<< KnotTheory`
<< bbCalculus.m
```

```
Loading KnotTheory` version of August 22, 2010, 13:36:57.55.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
b = Rp[2, 3] ** Rp[1, 3] ** Rp[1, 2] ** Rm[2, 1] ** Rp[3, 2]
```

$$\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & \frac{-1+T_1}{T_2} & \frac{-1+T_1}{T_2} \\ t[2] & \left(-1+\frac{1}{T_2}\right) T_3 & (1-T_1)\left(-1+\frac{1}{T_2}\right) T_3 & \frac{(-1+T_2)\,(-1+T_1\,(1+T_2))\,T_3}{T_2} \\ t[3] & \left(1-\frac{1}{T_2}\right)(-1+T_3) & \frac{(-1+T_1+T_2)\,(-1+T_3)}{T_2} & \frac{(1-T_2)\,(-1+T_1\,(1+T_2))\,(-1+T_3)}{T_2} \\ 1+\Sigma/\omega & \frac{1}{T_2} & T_1\,T_3 & T_1\,T_2 \end{pmatrix}$$

```
CT1[B[ω_, σ_, μ_]] := Simplify[ω (σ - Total[h /@ hL[σ]]) == (μ /. _t → 1)]
```

```
CT1[b]
```

```
True
```

```
CT2[B[ω_, _, μ_]] := Module[
  {μ1, tails, heads, mat},
  μ1 = μ /. T_ → T;
  {tails, heads} = {tL[μ1], hL[μ1]};
  If[Length[tails] < 2 || Length[heads] < 2, True,
   mat = Outer[(∂t[#1],h[#2] μ1) &, tL[μ1], hL[μ1]];
   And @@ ((1 == Denominator[Factor[T^1000 # / ω]]) & /@ Flatten[Minors[mat, 2]])
  ]
 ]
```

```
CT2[b]
```

```
True
```

```
BB @@ b /. T_ → T
```

$$\begin{aligned} &BB\Big[1,\ \frac{h[1]}{T}+T^2\,h[2]+T^2\,h[3],\ h[1]\left(\left(-1+\frac{1}{T}\right)T\,t[2]+\left(1-\frac{1}{T}\right)(-1+T)\,t[3]\right)+ \\ &\quad h[2]\left(\frac{(-1+T)\,t[1]}{T}+\left(-1+\frac{1}{T}\right)(1-T)\,T\,t[2]+\frac{(-1+T)\,(-1+2\,T)\,t[3]}{T}\right)+ \\ &\quad h[3]\left(\frac{(-1+T)\,t[1]}{T}+(-1+T)\,(-1+T\,(1+T))\,t[2]+\frac{(1-T)\,(-1+T)\,(-1+T\,(1+T))\,t[3]}{T}\right)\Big] \end{aligned}$$

```
CT2[
  B[T - 2, h[1]/T + T^2 h[2] + T^2 h[3], h[1] ((-1 + 1/T) T t[2] + (1 - 1/T) (-1 + T) t[3]) +
     h[2] ((-1 + T) t[1]/T + (-1 + 1/T) (1 - T) T t[2] + (-1 + T) (-1 + 2 T) t[3]/T) + h[3]
      ((-1 + T) t[1]/T + (-1 + T) (-1 + T (1 + T)) t[2] + (1 - T) (-1 + T) (-1 + T (1 + T)) t[3]/T)]
]
```

```
1 == -2 + T && 1 == -2 + T && 1 == -2 + T &&
 1 == -2 + T && 1 == -2 + T && 1 == -2 + T && 1 == -2 + T && 1 == -2 + T
```

```
CT[b_B] := CT1[b] && CT2[b]
```

```
CT[
  B[T - 2, h[1]/T + T^2 h[2] + T^2 h[3], h[1] ((-1 + 1/T) T t[2] + (1 - 1/T) (-1 + T) t[3]) +
     h[2] ((-1 + T) t[1]/T + (-1 + 1/T) (1 - T) T t[2] + (-1 + T) (-1 + 2 T) t[3]/T) + h[3]
      ((-1 + T) t[1]/T + (-1 + T) (-1 + T (1 + T)) t[2] + (1 - T) (-1 + T) (-1 + T (1 + T)) t[3]/T)]
]
```

```
(3 - 4 T + T^2) (-h[1] + T (1 + T) (h[2] + h[3]))/T == 0 && 1 == -2 + T && 1 == -2 + T &&
 1 == -2 + T && 1 == -2 + T && 1 == -2 + T && 1 == -2 + T && 1 == -2 + T
```

```
CT[b]
```

```
True
```

```
Test[K_] := Module[
  {b, ok},
  b = Times @@ (PD[K] /.
      X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]], Rp[l, i], Rm[j, i]]);
  b = b /. T_ -> T;
  ok = CT2[b];
  Do[b = dm[1, k, 1][b]; ok = ok && CT2[b], {k, 2, 2 Crossings[K]}];
  {ok, b[[1]], Alexander[K][T]}
 ]
```

```
Test[Knot[10, 165]]
```

```
KnotTheory::loading : Loading precomputed data in PD4Knots`.
```

$$\left\{\text{True}, T^2 \left(-2 + 10\,T - 15\,T^2 + 10\,T^3 - 2\,T^4\right), -15 - \frac{2}{T^2} + \frac{10}{T} + 10\,T - 2\,T^2\right\}$$

```
(# → Test[#][[1]]) & /@ AllKnots[7]
```

```
{Knot[7, 1] → True, Knot[7, 2] → True, Knot[7, 3] → True,
 Knot[7, 4] → True, Knot[7, 5] → True, Knot[7, 6] → True, Knot[7, 7] → True}
```

```
Analyze[K_] := Module[
  {b, ok},
  b = Times @@ (PD[K] /.
      X[i_, j_, k_, l_] :↦ If[PositiveQ[X[i, j, k, l]], Rp[l, i], Rm[j, i]]);
  b = b /. T_ → T;
  ok = CT2[b];
  Do[
   b = dm[1, k, 1][b];
   ok = ok && CT2[b];
   Print[{k, b, ok}],
   {k, 2, 2 Crossings[K]}];
  {ok, b[[1]], Alexander[K][T]}
 ]
```

```
False && Analyze[Knot[7, 7]]
```

```
False
```

```
bbSimplify = Expand[Factor[#]] &; n = 10;
b = Product[If[Random[] < 1, Rp[i, i], Rm[i, i]], {i, n}];
b = b /. T_ → T;
ts = hs = Range[n];
Print[β[k = 0] = b];
While[CT2[b] && Length[ts] + Length[hs] > 2,
  i = ts[[ RandomInteger[{1, Length[ts]}] ]];
  j = hs[[ RandomInteger[{1, Length[hs]}] ]];
  r = Random[];
  Which[
   (r -= 0.6) < 0, (
    Print[{++k, {ts, hs}, "swapth", {i, j}}];
    Print[β[k] = b = b // swapth[i, j]]
   ),
   (r -= 0.2) < 0, (
    j = ts[[ RandomInteger[{1, Length[ts]}] ]];
    If[i ≠ j,
     Print[{++k, {ts, hs}, "tm", {i, j, i}}];
     Print[β[k] = b = b // tm[i, j, i]];
     ts = DeleteCases[ts, j]
    ]
   ),
   True, (
    i = hs[[ RandomInteger[{1, Length[hs]}] ]];
    If[i ≠ j,
     Print[{++k, {ts, hs}, "hm", {i, j, i}}];
     Print[β[k] = b = b // hm[i, j, i]];
     hs = DeleteCases[hs, j]
    ]
   )
  ]
 ];
{CT2[b], b}
```

	1	h[1]	h[2]	h[3]	h[4]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]
t[1]	$-1+T$	0	0	0	0	0	0	0	0	0	
t[2]	0	$-1+T$	0	0	0	0	0	0	0	0	
t[3]	0	0	$-1+T$	0	0	0	0	0	0	0	
t[4]	0	0	0	$-1+T$	0	0	0	0	0	0	
t[5]	0	0	0	0	$-1+T$	0	0	0	0	0	
t[6]	0	0	0	0	0	$-1+T$	0	0	0	0	
t[7]	0	0	0	0	0	0	$-1+T$	0	0	0	
t[8]	0	0	0	0	0	0	0	$-1+T$	0	0	
t[9]	0	0	0	0	0	0	0	0	$-1+T$	0	
t[10]	0	0	0	0	0	0	0	0	0	$-1+T$	
$1+\Sigma/\omega$	T	T	T	T	T	T	T	T	T	T	

```
{1, {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}}, swapth, {7, 6}}
```

	h[1]	h[2]	h[3]	h[4]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]
t[1]	$-1+T$	0	0	0	0	0	0	0	0	0
t[2]	0	$-1+T$	0	0	0	0	0	0	0	0
t[3]	0	0	$-1+T$	0	0	0	0	0	0	0
t[4]	0	0	0	$-1+T$	0	0	0	0	0	0
t[5]	0	0	0	0	$-1+T$	0	0	0	0	0
t[6]	0	0	0	0	0	$-1+T$	$-1+2T-T^2$	0	0	0
t[7]	0	0	0	0	0	0	$-T+T^2$	0	0	0
t[8]	0	0	0	0	0	0	0	$-1+T$	0	0
t[9]	0	0	0	0	0	0	0	0	$-1+T$	0
t[10]	0	0	0	0	0	0	0	0	0	$-1+T$
$1+\Sigma/\omega$	T	T	T	T	T	T	T	T	T	T

{2, {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}}, swapth, {3, 9}}

	h[1]	h[2]	h[3]	h[4]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]
t[1]	$-1+T$	0	0	0	0	0	0	0	0	0
t[2]	0	$-1+T$	0	0	0	0	0	0	0	0
t[3]	0	0	$-T+T^2$	0	0	0	0	0	0	0
t[4]	0	0	0	$-1+T$	0	0	0	0	0	0
t[5]	0	0	0	0	$-1+T$	0	0	0	0	0
t[6]	0	0	0	0	0	$-1+T$	$-1+2T-T^2$	0	0	0
t[7]	0	0	0	0	0	0	$-T+T^2$	0	0	0
t[8]	0	0	0	0	0	0	0	$-1+T$	0	0
t[9]	0	0	$-1+2T-T^2$	0	0	0	0	0	$-1+T$	0
t[10]	0	0	0	0	0	0	0	0	0	$-1+T$
$1+\Sigma/\omega$	T	T	T	T	T	T	T	T	T	T

{3, {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}}, hm, {1, 4, 1}}

	h[1]	h[2]	h[3]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]
t[1]	$-1+T$	0	0	0	0	0	0	0	0
t[2]	0	$-1+T$	0	0	0	0	0	0	0
t[3]	0	0	$-T+T^2$	0	0	0	0	0	0
t[4]	$-T+T^2$	0	0	0	0	0	0	0	0
t[5]	0	0	0	$-1+T$	0	0	0	0	0
t[6]	0	0	0	0	$-1+T$	$-1+2T-T^2$	0	0	0
t[7]	0	0	0	0	0	$-T+T^2$	0	0	0
t[8]	0	0	0	0	0	0	$-1+T$	0	0
t[9]	0	0	$-1+2T-T^2$	0	0	0	0	$-1+T$	0
t[10]	0	0	0	0	0	0	0	0	$-1+T$
$1+\Sigma/\omega$	T^2	T	T	T	T	T	T	T	T

{4, {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 7, 8, 9, 10}}, swapth, {6, 3}}

	h[1]	h[2]	h[3]	h[5]	h[6]	h[7]	h[8]	h[9]	h[
t[1]	$-1+T$	0	0	0	0	0	0	0	
t[2]	0	$-1+T$	0	0	0	0	0	0	
t[3]	0	0	$-T+T^2$	0	$-T+2T^2-T^3$	$-T+3T^2-3T^3+T^4$	0	0	
t[4]	$-T+T^2$	0	0	0	0	0	0	0	
t[5]	0	0	0	$-1+T$	0	0	0	0	
t[6]	0	0	0	0	$-T+T^2$	$-T+2T^2-T^3$	0	0	
t[7]	0	0	0	0	0	$-T+T^2$	0	0	
t[8]	0	0	0	0	0	0	$-1+T$	0	
t[9]	0	0	$-1+2T-T^2$	0	$-1+3T-3T^2+T^3$	$-1+4T-6T^2+4T^3-T^4$	0	$-1+T$	
t[10]	0	0	0	0	0	0	0	0	-1
$1+\Sigma/\omega$	T^2	T	T	T	T	T	T	T	

{5, {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 7, 8, 9, 10}}, tm, {6, 1, 6}}

	1	h[1]	h[2]	h[3]	h[5]	h[6]	h[7]	h[8]	h[9]	h[
t[2]	0	$-1+T$	0	0	0	0	0	0	0	
t[3]	0	0	$-T+T^2$	0	$-T+2T^2-T^3$	$-T+3T^2-3T^3+T^4$	0	0		
t[4]	$-T+T^2$	0	0	0	0	0	0	0		
t[5]	0	0	0	$-1+T$	0	0	0	0		
t[6]	$-1+T$	0	0	0	$-T+T^2$	$-T+2T^2-T^3$	0	0		
t[7]	0	0	0	0	0	$-T+T^2$	0	0		
t[8]	0	0	0	0	0	0	$-1+T$	0		
t[9]	0	0	$-1+2T-T^2$	0	$-1+3T-3T^2+T^3$	$-1+4T-6T^2+4T^3-T^4$	0	$-1+T$		
t[10]	0	0	0	0	0	0	0	0	-1	
$1+\Sigma/\omega$	T^2	T	T	T	T	T	T	T		

{6, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 7, 8, 9, 10}}, hm, {10, 7, 10}}

	1	h[1]	h[2]	h[3]	h[5]	h[6]	h[8]	h[9]	h[10]
t[2]	0	$-1+T$	0	0	0	0	0	0	0
t[3]	0	0	$-T+T^2$	0	$-T+2T^2-T^3$	0	0	$-T^2+3T^3-3T^4+T^5$	
t[4]	$-T+T^2$	0	0	0	0	0	0	0	
t[5]	0	0	0	$-1+T$	0	0	0	0	
t[6]	$-1+T$	0	0	0	$-T+T^2$	0	0	$-T^2+2T^3-T^4$	
t[7]	0	0	0	0	0	0	0	$-T^2+T^3$	
t[8]	0	0	0	0	0	$-1+T$	0	0	
t[9]	0	0	$-1+2T-T^2$	0	$-1+3T-3T^2+T^3$	0	$-1+T$	$-T+4T^2-6T^3+4T^4-T^5$	
t[10]	0	0	0	0	0	0	0	$-1+T$	
$1+\Sigma/\omega$	T^2	T	T	T	T	T	T	T^2	

{7, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 8, 9, 10}}, swapth, {8, 6}}

	1	h[1]	h[2]	h[3]	h[5]	h[6]	h[8]	h[9]	h
t[2]	0	$-1+T$	0	0	0	0	0	0	
t[3]	0	0	$-T+T^2$	0	$-T+2T^2-T^3$	$-T+3T^2-3T^3+T^4$	0	$-T^2+3$	
t[4]	$-T+T^2$	0	0	0	0	0	0	0	
t[5]	0	0	0	$-1+T$	0	0	0	0	
t[6]	$-1+T$	0	0	0	$-T+T^2$	$-T+2T^2-T^3$	0	$-T^2$	
t[7]	0	0	0	0	0	0	0	$-$	
t[8]	0	0	0	0	0	$-T+T^2$	0		
t[9]	0	0	$-1+2T-T^2$	0	$-1+3T-3T^2+T^3$	$-1+4T-6T^2+4T^3-T^4$	$-1+T$	$-T+4T^2-$	
t[10]	0	0	0	0	0	0	0		
$1+\Sigma/\omega$	T^2	T	T	T	T	T	T		

{8, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 8, 9, 10}}, swapth, {10, 6}}

	1	h[1]	h[2]	h[3]	h[5]	h[6]	h[8]	h[9]	
t[2]	0	$-1+T$	0	0	0	0	0	0	
t[3]	0	0	$-T+T^2$	0	$-T+2T^2-T^3$	$-T+3T^2-3T^3+T^4$	0	$-T+$	
t[4]	$-T+T^2$	0	0	0	0	0	0	0	
t[5]	0	0	0	$-1+T$	0	0	0	0	
t[6]	$-1+T$	0	0	0	$-T+T^2$	$-T+2T^2-T^3$	0	$-T$	
t[7]	0	0	0	0	0	0	0	0	
t[8]	0	0	0	0	0	$-T+T^2$	0		
t[9]	0	0	$-1+2T-T^2$	0	$-1+3T-3T^2+T^3$	$-1+4T-6T^2+4T^3-T^4$	$-1+T$	$-1+3T-$	
t[10]	0	0	0	0	0	0	0		
$1+\Sigma/\omega$	T^2	T	T	T	T	T	T		

{9, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 8, 9, 10}}, hm, {9, 8, 9}}

	1	h[1]	h[2]	h[3]	h[5]	h[6]	h[9]	h[10
t[2]	0	$-1+T$	0	0	0	0	0	0
t[3]	0	0	$-T+T^2$	0	$-T+2T^2-T^3$	$-T^2+3T^3-3T^4+T^5$	$-T+2T^2-$	
t[4]	$-T+T^2$	0	0	0	0	0	0	0
t[5]	0	0	0	$-1+T$	0	0	0	0
t[6]	$-1+T$	0	0	0	$-T+T^2$	$-T^2+2T^3-T^4$	$-T+T^2+$	
t[7]	0	0	0	0	0	0	$-T^2+$	
t[8]	0	0	0	0	0	$-T^2+T^3$	0	
t[9]	0	0	$-1+2T-T^2$	0	$-1+3T-3T^2+T^3$	$-1+4T^2-6T^3+4T^4-T^5$	$-1+3T-2T^2-$	
t[10]	0	0	0	0	0	0	$-T+$	
$1+\Sigma/\omega$	T^2	T	T	T	T	T^2	T^2	

{10, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 9, 10}}, swapth, {5, 9}}

	1	h[1]	h[2]	h[3]	h[5]	h[6]	h[
t[2]	0	$-1+T$	0	0	0	0	
t[3]	0	0	$-T+T^2$	$-T^2+4T^3-6T^4+4T^5-T^6$	$-T+2T^2-T^3$	$-T^2+3T^3$	
t[4]	$-T+T^2$	0	0	0	0	0	
t[5]	0	0	0	$-T^2+T^3$	0		
t[6]	$-1+T$	0	0	$-T^2+3T^3-3T^4+T^5$	$-T+T^2$	$-T^2+2$	
t[7]	0	0	0	0	0		
t[8]	0	0	0	$-T^2+2T^3-T^4$	0	$-T^2$	
t[9]	0	0	$-1+2T-T^2$	$-1+T+4T^2-10T^3+10T^4-5T^5+T^6$	$-1+3T-3T^2+T^3$	$-1+4T^2-6$	
t[10]	0	0	0	0	0	0	
$1+\Sigma/\omega$	T^2	T	T	T	T	T	

{11, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 9, 10}}, swapth, {3, 9}}

	$1-T^2+3T^3-3T^4+T^5$	h[1]	h[2]	h[3]
t[2]		0	$-1+T+T^2-4T^3+6T^4-4T^5+T^6$	0
t[3]		0	0	$-T^3+T^4$
t[4]		$-T+T^2+T^3-4T^4+6T^5-4T^6+T^7$	0	0
t[5]		0	0	0
t[6]		$-1+T+T^2-4T^3+6T^4-4T^5+T^6$	0	$-T^3+3T^4-3T$
t[7]		0	0	0
t[8]		0	0	$-T^3+2T^4-$
t[9]		0	0	$-1+T+T^2-$
t[10]		0	0	0
$1+\Sigma/\omega$		T^2	T	T

{12, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 9, 10}}, swapth, {4, 9}}

	$1-T^2+3T^3-3T^4+T^5$	h[1]	h[2]	h[3
t[2]		0	$-1+T+T^2-4T^3+6T^4-4T^5+T^6$	0
t[3]		$-T^5+4T^6-6T^7+4T^8-T^9$	0	$-T^3-$
t[4]		$-T^3+T^4+T^5-4T^6+6T^7-4T^8+T^9$	0	0
t[5]		0	0	0
t[6]		$-1+T+T^2-5T^3+9T^4-7T^5+2T^6$	0	$-T^3+3T^4-$
t[7]		0	0	0
t[8]		$-T^3+2T^4-T^5$	0	$-T^3+2$
t[9]		$-T+T^2+4T^3-10T^4+10T^5-5T^6+T^7$	0	$-1+T+$
t[10]		0	0	0
$1+\Sigma/\omega$		T^2	T	T

{13, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 9, 10}}, swapth, {2, 10}}

$$\begin{pmatrix} 1 - T^2 + 3T^3 - 3T^4 + T^5 & h[1] & h[2] \\ t[2] & 0 & -T^2 + T^3 + T^4 - 4T^5 + 6T^6 - 4T^7 + T^8 \\ t[3] & -T^5 + 4T^6 - 6T^7 + 4T^8 - T^9 & -T^3 + 3T^4 - 2T^5 - 2T^6 + 3T^7 - T^8 \\ t[4] & -T^3 + T^4 + T^5 - 4T^6 + 6T^7 - 4T^8 + T^9 & 0 \\ t[5] & 0 & 0 \\ t[6] & -1 + T + T^2 - 5T^3 + 9T^4 - 7T^5 + 2T^6 & -T + 2T^2 - 2T^4 + T^5 \\ t[7] & 0 & -T^2 + 2T^3 - 5T^5 + 10T^6 - 10T^7 + 5T^8 - T^9 \\ t[8] & -T^3 + 2T^4 - T^5 & -T^3 + 4T^4 - 5T^5 + 5T^7 - 4T^8 + T^9 \\ t[9] & -T + T^2 + 4T^3 - 10T^4 + 10T^5 - 5T^6 + T^7 & -1 + 3T - T^2 - 5T^3 + 5T^4 + T^5 - 3T^6 + T^7 \\ t[10] & 0 & -T + 2T^2 - 5T^4 + 10T^5 - 10T^6 + 5T^7 - T^8 \\ 1_{+\Sigma/\omega} & T^2 & T \end{pmatrix}$$

$$\{14, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 2, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{8, 2\}\}$$

$$\begin{pmatrix} 1 - T^2 + 2T^3 + T^4 - 4T^5 + 5T^7 - 4T^8 + T^9 & h[1] \\ t[2] & -T^5 + 3T^6 - 3T^7 + T^8 \\ t[3] & -T^5 + 3T^6 - T^7 - 5T^8 + 4T^9 + 5T^{10} - 9T^{11} + 5T^{12} - T^{13} \\ t[4] & -T^3 + T^4 + T^5 - 3T^6 + T^7 + 5T^8 - 4T^9 - 5T^{10} + 9T^{11} - 5T^{12} + T^{13} \\ t[5] & 0 \\ t[6] & -1 + T + T^2 - 4T^3 + 3T^4 + 6T^5 - 8T^6 - 5T^7 + 14T^8 - 9T^9 + 2T^{10} \\ t[7] & -T^5 + 4T^6 - 6T^7 + 4T^8 - T^9 \\ t[8] & -T^4 + 2T^5 - T^6 \\ t[9] & -T + T^2 + 3T^3 - 4T^4 - 4T^5 + 9T^6 + T^7 - 14T^8 + 14T^9 - 6T^{10} + T^{11} \\ t[10] & -T^4 + 4T^5 - 6T^6 + 4T^7 - T^8 \\ 1_{+\Sigma/\omega} & T^2 \end{pmatrix}$$

$$\{15, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 2, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{4, 6\}\}$$

$$\begin{pmatrix} 1 - T^2 + 2T^3 + T^4 - 4T^5 + 5T^7 - 4T^8 + T^9 & h[1] \\ t[2] & -T^5 + 3T^6 - 3T^7 + 5T^9 - 10T^{10} + 10T^{11} - 5T^{12} + T^{13} \\ t[3] & -T^5 + 2T^6 + 2T^7 - 8T^8 + 5T^9 + 5T^{10} - 9T^{11} + 5T^{12} - T^{13} \\ t[4] & -T^4 + T^5 + T^6 - 3T^7 + T^8 + 5T^9 - 4T^{10} - 5T^{11} + 9T^{12} - 5T^{13} + T^{14} \\ t[5] & 0 \\ t[6] & -1 + T + T^2 - 4T^3 + 2T^4 + 8T^5 - 9T^6 - 5T^7 + 14T^8 - 9T^9 + 2T^{10} \\ t[7] & -T^5 + 4T^6 - 6T^7 + 3T^8 + 5T^9 - 15T^{10} + 20T^{11} - 15T^{12} + 6T^{13} - T^{14} \\ t[8] & -T^4 + 2T^5 - T^6 - T^7 + 4T^8 - 6T^9 + 4T^{10} - T^{11} \\ t[9] & -T + T^2 + 2T^3 - T^4 - 6T^5 + 7T^6 + 4T^7 - 15T^8 + 14T^9 - 6T^{10} + T^{11} \\ t[10] & -T^4 + 4T^5 - 6T^6 + 3T^7 + 5T^8 - 15T^9 + 20T^{10} - 15T^{11} + 6T^{12} - T^{13} \\ 1_{+\Sigma/\omega} & T^2 \end{pmatrix}$$

$$\{16, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 2, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{9, 10\}\}$$

$$\begin{pmatrix} 2T - 2T^3 + 2T^4 - 2T^5 - T^6 + 5T^7 - 4T^8 + T^9 & h[1] \\ t[2] & -3T^6 + 11T^7 - 14T^8 + 5T^9 + 4T^{10} - 4T^{11} + T^{12} \\ t[3] & -T^4 + 3T^5 - 3T^6 + 2T^7 - 3T^8 + T^9 + 6T^{10} - 9T^{11} + 5T^{12} - \\ t[4] & -2T^5 + 2T^6 + 2T^7 - 4T^8 + 4T^9 - T^{10} - 6T^{11} + 9T^{12} - 5T^{13} \\ t[5] & 0 \\ t[6] & -2T + T^2 + 4T^3 - 5T^4 + 5T^5 - 3T^6 - 7T^7 + 14T^8 - 9T^9 + 2 \\ t[7] & -T^3 + 2T^4 + T^5 - 6T^6 + 9T^7 - 12T^8 + 16T^9 - 20T^{10} + 21T^{11} - 15T^{1} \\ t[8] & -3T^5 + 8T^6 - 6T^7 - T^8 + 3T^9 - T^{10} \\ t[9] & -T^3 + T^4 + 2T^5 - T^6 - 6T^7 + 7T^8 + 4T^9 - 15T^{10} + 14T^{11} - 6T^{1} \\ t[10] & -T^2 + 2T^3 + T^4 - 6T^5 + 9T^6 - 12T^7 + 16T^8 - 20T^9 + 21T^{10} - 15T^{11} \\ 1_{+\Sigma/\omega} & T^2 \end{pmatrix}$$

$$\{17, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 2, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{5, 5\}\}$$

$$\begin{pmatrix} 2\,T-4\,T^3+4\,T^4-5\,T^6+9\,T^7-5\,T^8-5\,T^9+9\,T^{10}-5\,T^{11}+T^{12} & h[\\ t[2] & -3\,T^6+11\,T^7-11\,T^8-9\,T^9+29\,T^{10} \\ t[3] & -T^4+3\,T^5-2\,T^6-2\,T^7+3\,T^8-4\,T^9+11 \\ t[4] & -2\,T^5+2\,T^6+4\,T^7-8\,T^8+4\,T^9+5\,T^{10}-1 \\ t[5] & (\\ t[6] & -2\,T+T^2+6\,T^3-8\,T^4+2\,T^5+6\,T^6-17\,T^7+ \\ t[7] & -T^3+2\,T^4+2\,T^5-9\,T^6+10\,T^7-5\,T^8+T^9+T^{10}-7 \\ t[8] & -3\,T^5+8\,T^6-3\,T^7-12\,T^8+17 \\ t[9] & -T^3+T^4+3\,T^5-3\,T^6-7\,T^7+10\,T^8+9\,T^9-28\,T \\ t[10] & -T^2+2\,T^3+2\,T^4-9\,T^5+10\,T^6-5\,T^7+T^8+T^9-7 \\ 1+\Sigma/\omega & T \end{pmatrix}$$

{18, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 9, 10}}, swapth, {4, 3}}

$$\begin{pmatrix} 2\,T-4\,T^3+4\,T^4-5\,T^6+9\,T^7-5\,T^8-5\,T^9+9\,T^{10}-5\,T^{11}+T^{12} & \\ t[2] & -3\,T^6+11\,T^7-11\,T^8-10\,T^9+3 \\ t[3] & -T^4+3\,T^5-2\,T^6-3\,T^7+5\,T^8-4\,T^9 \\ t[4] & -2\,T^6+2\,T^7+4\,T^8-8\,T^9+4\,T^{10}+5 \\ t[5] & \\ t[6] & -2\,T+T^2+6\,T^3-8\,T^4+T^5+9\,T^6-18\,T^7+16\,T^8+ \\ t[7] & -T^3+2\,T^4+2\,T^5-10\,T^6+13\,T^7-6\,T^8-6 \\ t[8] & -3\,T^5+8\,T^6-3\,T^7-13\,T^8 \\ t[9] & -T^3+T^4+3\,T^5-4\,T^6-5\,T^7+11\,T^8+4\,T \\ t[10] & -T^2+2\,T^3+2\,T^4-10\,T^5+13\,T^6-6\,T^7-6 \\ 1+\Sigma/\omega & \end{pmatrix}$$

{19, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 9, 10}}, swapth, {4, 3}}

$$\begin{pmatrix} 2\,T-4\,T^3+4\,T^4-5\,T^6+9\,T^7-5\,T^8-5\,T^9+9\,T^{10}-5\,T^{11}+T^{12} & \\ t[2] & -3\,T^6+11\,T^7-11\,T^8-10\,T^9+32 \\ t[3] & -T^4+3\,T^5-2\,T^6-3\,T^7+4\,T^8-2 \\ t[4] & -2\,T^7+2\,T^8+4\,T^9-8\,T^{10}+4\,T^{1} \\ t[5] & \\ t[6] & -2\,T+T^2+6\,T^3-8\,T^4+T^5+8\,T^6-15\,T^7+15\,T^8- \\ t[7] & -T^3+2\,T^4+2\,T^5-10\,T^6+12\,T^7-3\,T^8-7\,T^9+ \\ t[8] & -3\,T^5+8\,T^6-3\,T^7-13\,T^{8} \\ t[9] & -T^3+T^4+3\,T^5-4\,T^6-6\,T^7+13\,T^8+ \\ t[10] & -T^2+2\,T^3+2\,T^4-10\,T^5+12\,T^6-3\,T^7-7\,T^8- \\ 1+\Sigma/\omega & \end{pmatrix}$$

{20, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 5, 6, 9, 10}}, hm, {1, 2, 1}}

$$\begin{pmatrix} 2\,T-4\,T^3+4\,T^4-5\,T^6+9\,T^7-5\,T^8-5\,T^9+9\,T^{10}-5\,T^{11}+T^{12} & \\ t[2] & -2\,T^5-T^6+14\,T^7-14\,T^8-14\,T^9+ \\ t[3] & -T^4+2\,T^5+T^6-4\,T^7-2\,T^8+6 \\ t[4] & -2\,T^7+2\,T^8+4\,T^9-8\,T^{10}+4\,T^{1} \\ t[5] & \\ t[6] & -2\,T+T^2+5\,T^3-6\,T^4+2\,T^5+3\,T^6-12\,T^7+17\,T^8 \\ t[7] & -T^3+T^4+4\,T^5-9\,T^6+4\,T^7+9\,T^8-12\,T^9- \\ t[8] & -3\,T^5+7\,T^6+T^7-17\,T^8+1 \\ t[9] & -T^3+6\,T^5-4\,T^6-15\,T^7+22\,T^8+1 \\ t[10] & -T^2+T^3+4\,T^4-9\,T^5+4\,T^6+9\,T^7-12\,T^8- \\ 1+\Sigma/\omega & \end{pmatrix}$$

{21, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 3, 5, 6, 9, 10}}, swapth, {10, 6}}

$$\begin{pmatrix} T + 3\,T^2 - 5\,T^3 - 3\,T^4 + 13\,T^5 - 15\,T^6 + 8\,T^7 + 9\,T^8 - 25\,T^9 + 24\,T^{10} - 11\,T^{11} + 2\,T^{12} & \\ t[2] & -T^5 - 4 \\ t[3] & -T^4 + \\ t[4] & -T^7 - 2 \\ t[5] & \\ t[6] & -T - 3\,T^2 + 8\,T^3 + 3\,T^4 - 21\,T^5 + \\ t[7] & -T^3 + T^4 + 4\,T^5 \\ t[8] & \\ t[9] & -T^3 + 5 \\ t[10] & -T^3 + T^4 + 4\,T^5 \\ 1+\Sigma/\omega & \end{pmatrix}$$

$$\{22, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{9, 9\}\}$$

$$\begin{pmatrix} T + 2\,T^2 - 5\,T^3 + T^4 + 10\,T^5 - 17\,T^6 + 2\,T^7 + 31\,T^8 - 37\,T^9 - T^{10} + 36\,T^{11} - 33\,T^{12} + 13\,T^{13} - 2\,T^{14} & \\ t[2] & \\ t[3] & \\ t[4] & \\ t[5] & \\ t[6] & -T - 2\,T^2 + 7\,T^3 - \\ t[7] & -T \\ t[8] & \\ t[9] & \\ t[10] & -T \\ 1+\Sigma/\omega & \end{pmatrix}$$

$$\{23, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{8, 5\}\}$$

$$\begin{pmatrix} T + 2\,T^2 - 5\,T^3 - T^4 + 12\,T^5 - 7\,T^6 - 23\,T^7 + 55\,T^8 - 48\,T^9 + T^{10} + 36\,T^{11} - 33\,T^{12} + 13\,T^{13} - 2\,T^{14} & \\ t[2] & \\ t[3] & \\ t[4] & \\ t[5] & \\ t[6] & -T - 2\,T^2 + 7\,T^3 + \\ t[7] & - \\ t[8] & \\ t[9] & \\ t[10] & - \\ 1+\Sigma/\omega & \end{pmatrix}$$

$$\{24, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{9, 10\}\}$$

$$\begin{pmatrix} T + 2\,T^2 - 5\,T^3 - 2\,T^4 + 13\,T^5 - 2\,T^6 - 30\,T^7 + 47\,T^8 - 30\,T^9 + 4\,T^{10} + 5\,T^{11} - 2\,T^{12} & \\ t[2] & -T^5 - 3\,T^6 + 10\,T^7 + 9\,T^8 \\ t[3] & -T^4 + T^5 + 5 \\ t[4] & -T^7 - T^8 \\ t[5] & -2\,T^8 + 3 \\ t[6] & -T - 2\,T^2 + 7\,T^3 + 2\,T^4 - 19\,T^5 + \\ t[7] & -T^3 + T^4 + 4\,T^5 - 8 \\ t[8] & -2\,T^6 + T^7 + \\ t[9] & -T^7 + 5\,T^9 - \\ t[10] & -T^3 + T^4 + 4\,T^5 - 8 \\ 1+\Sigma/\omega & \end{pmatrix}$$

$$\{25, \{\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1, 3, 5, 6, 9, 10\}\}, \text{swapth}, \{3, 1\}\}$$

$$\left(\begin{array}{l} T + 2\,T^2 - 5\,T^3 - 3\,T^4 + 14\,T^5 + 3\,T^6 - 39\,T^7 + 48\,T^8 - 33\,T^9 + 31\,T^{10} - 17\,T^{11} - 46\,T^{12} + 108\,T^{13} - 104\,T^{14} + 55\,T^{15} - \\ t[2] \\ t[3] \\ t[4] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ t[10] \\ 1+\Sigma/\omega \end{array}\right.$$

{26, {{2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 3, 5, 6, 9, 10}}, tm, {3, 10, 3}}

$$\left(\begin{array}{l} T + 2\,T^2 - 5\,T^3 - 3\,T^4 + 14\,T^5 + 3\,T^6 - 39\,T^7 + 48\,T^8 - 33\,T^9 + 31\,T^{10} - 17\,T^{11} - 46\,T^{12} + 108\,T^{13} - 104\,T^{14} + 55\,T^{15} - \\ t[2] \\ t[3] \\ t[4] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{array}\right.$$

{27, {{2, 3, 4, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9, 10}}, tm, {8, 3, 8}}

$$\left(\begin{array}{l} T + 2\,T^2 - 5\,T^3 - 3\,T^4 + 14\,T^5 + 3\,T^6 - 39\,T^7 + 48\,T^8 - 33\,T^9 + 31\,T^{10} - 17\,T^{11} - 46\,T^{12} + 108\,T^{13} - 104\,T^{14} + 55\,T^{15} - \\ t[2] \\ t[4] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{array}\right.$$

{28, {{2, 4, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9, 10}}, swapth, {9, 6}}

$$\left(\begin{array}{l} T + 2\,T^2 - 5\,T^3 - 3\,T^4 + 14\,T^5 + 2\,T^6 - 37\,T^7 + 49\,T^8 - 37\,T^9 + 28\,T^{10} - T^{11} - 64\,T^{12} + 113\,T^{13} - 96\,T^{14} + 45\,T^{15} - 11\,T \\ t[2] \\ t[4] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{array}\right.$$

{29, {{2, 4, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9, 10}}, swapth, {8, 9}}

$$\begin{pmatrix} T + T^2 - 4\,T^3 + T^4 + 3\,T^5 + 5\,T^6 - 17\,T^7 + 30\,T^8 - 50\,T^9 + 54\,T^{10} - 17\,T^{11} - 36\,T^{12} + 76\,T^{13} - 108\,T^{14} + 128\,T^{15} - 119\,T \\ t[2] \\ t[4] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{pmatrix}$$

{30, {{2, 4, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9, 10}}, tm, {7, 4, 7}}

$$\begin{pmatrix} T + T^2 - 4\,T^3 + T^4 + 3\,T^5 + 5\,T^6 - 17\,T^7 + 30\,T^8 - 50\,T^9 + 54\,T^{10} - 17\,T^{11} - 36\,T^{12} + 76\,T^{13} - 108\,T^{14} + 128\,T^{15} - 119\,T \\ t[2] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{pmatrix}$$

{31, {{2, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9, 10}}, swapth, {8, 6}}

$$\begin{pmatrix} T + T^2 - 4\,T^3 + 6\,T^5 + 4\,T^6 - 24\,T^7 + 35\,T^8 - 28\,T^9 + 4\,T^{10} + 30\,T^{11} - 63\,T^{12} + 86\,T^{13} - 88\,T^{14} + 65\,T^{15} - 37\,T^{16} + 28\,T \\ t[2] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{pmatrix}$$

{32, {{2, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9, 10}}, swapth, {9, 9}}

$$\begin{pmatrix} T + T^2 - 4\,T^3 + 6\,T^5 + 4\,T^6 - 25\,T^7 + 35\,T^8 - 24\,T^9 + T^{10} + 28\,T^{11} - 68\,T^{12} + 104\,T^{13} - 94\,T^{14} + 39\,T^{15} + 2\,T^{16} - 15\,T^{18} \\ t[2] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{pmatrix}$$

{33, {{2, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9, 10}}, hm, {1, 10, 1}}

$$\begin{pmatrix} T + T^2 - 4\,T^3 + 6\,T^5 + 4\,T^6 - 25\,T^7 + 35\,T^8 - 24\,T^9 + T^{10} + 28\,T^{11} - 68\,T^{12} + 104\,T^{13} - 94\,T^{14} + 39\,T^{15} + 2\,T^{16} - 15\,T^{18} \\ t[2] \\ t[5] \\ t[6] \\ t[7] \\ t[8] \\ t[9] \\ 1+\Sigma/\omega \end{pmatrix}$$

{34, {{2, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9}}, swapth, {7, 1}}

$$\left(T + T^2 - 5\,T^3 + T^4 + 9\,T^5 - 3\,T^6 - 21\,T^7 + 34\,T^8 - 16\,T^9 - 3\,T^{10} - 7\,T^{11} + 3\,T^{12} + 53\,T^{13} - 92\,T^{14} + 49\,T^{15} + 24\,T^{16} - 57\right.$$

{35, {{2, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9}}, swapth, {2, 1}}

$$\left(T + T^2 - 5\,T^3 + T^4 + 8\,T^5 - 5\,T^6 - 13\,T^7 + 39\,T^8 - 49\,T^9 + 22\,T^{10} + 21\,T^{11} - 57\,T^{12} + 92\,T^{13} - 113\,T^{14} + 97\,T^{15} - 40\,T^{16}\right.$$

{36, {{2, 5, 6, 7, 8, 9}, {1, 3, 5, 6, 9}}, hm, {3, 1, 3}}

$$\left(T + T^2 - 5\,T^3 + T^4 + 8\,T^5 - 5\,T^6 - 13\,T^7 + 39\,T^8 - 49\,T^9 + 22\,T^{10} + 21\,T^{11} - 57\,T^{12} + 92\,T^{13} - 113\,T^{14} + 97\,T^{15} - 40\,T^{16}\right.$$

{37, {{2, 5, 6, 7, 8, 9}, {3, 5, 6, 9}}, swapth, {2, 6}}

$$\left(T + T^2 - 5\,T^3 + T^4 + 8\,T^5 - 5\,T^6 - 13\,T^7 + 39\,T^8 - 49\,T^9 + 21\,T^{10} + 22\,T^{11} - 49\,T^{12} + 73\,T^{13} - 106\,T^{14} + 116\,T^{15} - 55\,T^{1}\right.$$

{38, {{2, 5, 6, 7, 8, 9}, {3, 5, 6, 9}}, swapth, {5, 3}}

$$\left(T + T^2 - 5\,T^3 + T^4 + 8\,T^5 - 5\,T^6 - 14\,T^7 + 40\,T^8 - 46\,T^9 + 14\,T^{10} + 35\,T^{11} - 72\,T^{12} + 90\,T^{13} - 95\,T^{14} + 89\,T^{15} - 37\,T^{16}\right.$$

{39, {{2, 5, 6, 7, 8, 9}, {3, 5, 6, 9}}, tm, {5, 6, 5}}

$$T + T^2 - 5\,T^3 + T^4 + 8\,T^5 - 5\,T^6 - 14\,T^7 + 40\,T^8 - 46\,T^9 + 14\,T^{10} + 35\,T^{11} - 72\,T^{12} + 90\,T^{13} - 95\,T^{14} + 89\,T^{15} - 37\,T^{16}$$

$$\{40, \{\{2, 5, 7, 8, 9\}, \{3, 5, 6, 9\}\}, \text{swapth}, \{9, 5\}\}$$

$$T + T^2 - 5\,T^3 + T^4 + 8\,T^5 - 5\,T^6 - 14\,T^7 + 40\,T^8 - 47\,T^9 + 15\,T^{10} + 38\,T^{11} - 78\,T^{12} + 97\,T^{13} - 111\,T^{14} + 114\,T^{15} - 45\,T^{}$$

$$\{41, \{\{2, 5, 7, 8, 9\}, \{3, 5, 6, 9\}\}, \text{tm}, \{2, 5, 2\}\}$$

$$T + T^2 - 5\,T^3 + T^4 + 8\,T^5 - 5\,T^6 - 14\,T^7 + 40\,T^8 - 47\,T^9 + 15\,T^{10} + 38\,T^{11} - 78\,T^{12} + 97\,T^{13} - 111\,T^{14} + 114\,T^{15} - 45\,T^{}$$

$$\{42, \{\{2, 7, 8, 9\}, \{3, 5, 6, 9\}\}, \text{swapth}, \{8, 9\}\}$$

$$T + T^2 - 5\,T^3 + T^4 + 7\,T^5 - 4\,T^6 - 10\,T^7 + 29\,T^8 - 43\,T^9 + 33\,T^{10} + 16\,T^{11} - 72\,T^{12} + 95\,T^{13} - 126\,T^{14} + 181\,T^{15} - 125\,T^{}$$

$$\{43, \{\{2, 7, 8, 9\}, \{3, 5, 6, 9\}\}, \text{tm}, \{2, 7, 2\}\}$$

$$T + T^2 - 5\,T^3 + T^4 + 7\,T^5 - 4\,T^6 - 10\,T^7 + 29\,T^8 - 43\,T^9 + 33\,T^{10} + 16\,T^{11} - 72\,T^{12} + 95\,T^{13} - 126\,T^{14} + 181\,T^{15} - 125\,T^{}$$

$$\{44, \{\{2, 8, 9\}, \{3, 5, 6, 9\}\}, \text{swapth}, \{8, 3\}\}$$

$$T + T^2 - 5\,T^3 + T^4 + 7\,T^5 - 4\,T^6 - 11\,T^7 + 31\,T^8 - 43\,T^9 + 28\,T^{10} + 19\,T^{11} - 63\,T^{12} + 72\,T^{13} - 87\,T^{14} + 145\,T^{15} - 135\,T^{}$$

$$\{45, \{\{2, 8, 9\}, \{3, 5, 6, 9\}\}, \text{swapth}, \{9, 5\}\}$$

$$T + T^2 - 5\,T^3 + T^4 + 7\,T^5 - 4\,T^6 - 11\,T^7 + 31\,T^8 - 43\,T^9 + 27\,T^{10} + 20\,T^{11} - 60\,T^{12} + 66\,T^{13} - 80\,T^{14} + 129\,T^{15} - 110\,T^{}$$

{46, {{2, 8, 9}, {3, 5, 6, 9}}, swapth, {8, 5}}

$$T + T^2 - 5\,T^3 + T^4 + 7\,T^5 - 4\,T^6 - 11\,T^7 + 31\,T^8 - 43\,T^9 + 27\,T^{10} + 20\,T^{11} - 60\,T^{12} + 65\,T^{13} - 78\,T^{14} + 131\,T^{15} - 123\,T^{}$$

{47, {{2, 8, 9}, {3, 5, 6, 9}}, swapth, {8, 3}}

$$T + T^2 - 5\,T^3 + T^4 + 7\,T^5 - 4\,T^6 - 11\,T^7 + 31\,T^8 - 43\,T^9 + 27\,T^{10} + 20\,T^{11} - 60\,T^{12} + 65\,T^{13} - 79\,T^{14} + 133\,T^{15} - 123\,T^{}$$

{48, {{2, 8, 9}, {3, 5, 6, 9}}, swapth, {9, 3}}

$$T + T^2 - 5\,T^3 + T^4 + 7\,T^5 - 4\,T^6 - 11\,T^7 + 31\,T^8 - 43\,T^9 + 27\,T^{10} + 19\,T^{11} - 59\,T^{12} + 66\,T^{13} - 81\,T^{14} + 136\,T^{15} - 123\,T^{}$$

{49, {{2, 8, 9}, {3, 5, 6, 9}}, swapth, {2, 3}}

$$T^7 + T^8 - 5\,T^9 + T^{10} + 7\,T^{11} - 4\,T^{12} - 11\,T^{13} + 31\,T^{14} - 43\,T^{15} + 27\,T^{16} + 20\,T^{17} - 60\,T^{18} + 65\,T^{19} - 78\,T^{20} + 131\,T^{21} -$$

{50, {{2, 8, 9}, {3, 5, 6, 9}}, swapth, {9, 9}}

$$T^7 + T^8 - 5\,T^9 + T^{10} + 7\,T^{11} - 4\,T^{12} - 11\,T^{13} + 31\,T^{14} - 43\,T^{15} + 27\,T^{16} + 19\,T^{17} - 59\,T^{18} + 67\,T^{19} - 81\,T^{20} + 133\,T^{21} -$$

{51, {{2, 8, 9}, {3, 5, 6, 9}}, hm, {5, 3, 5}}

$$T^7 + T^8 - 5\,T^9 + T^{10} + 7\,T^{11} - 4\,T^{12} - 11\,T^{13} + 31\,T^{14} - 43\,T^{15} + 27\,T^{16} + 19\,T^{17} - 59\,T^{18} + 67\,T^{19} - 81\,T^{20} + 133\,T^{21} -$$

{52, {{2, 8, 9}, {5, 6, 9}}, swapth, {8, 6}}

$$T^7 + T^8 - 5\,T^9 + T^{10} + 7\,T^{11} - 4\,T^{12} - 11\,T^{13} + 31\,T^{14} - 43\,T^{15} + 27\,T^{16} + 19\,T^{17} - 59\,T^{18} + 67\,T^{19} - 82\,T^{20} + 136\,T^{21} -$$

{53, {{2, 8, 9}, {5, 6, 9}}, tm, {2, 8, 2}}

$$\left(T^7 + T^8 - 5\,T^9 + T^{10} + 7\,T^{11} - 4\,T^{12} - 11\,T^{13} + 31\,T^{14} - 43\,T^{15} + 27\,T^{16} + 19\,T^{17} - 59\,T^{18} + 67\,T^{19} - 82\,T^{20} + 136\,T^{21} -\right.$$

$$\{54, \{\{2, 9\}, \{5, 6, 9\}\}, \text{hm}, \{9, 6, 9\}\}$$

$$\left(T^7 + T^8 - 5\,T^9 + T^{10} + 7\,T^{11} - 4\,T^{12} - 11\,T^{13} + 31\,T^{14} - 43\,T^{15} + 27\,T^{16} + 19\,T^{17} - 59\,T^{18} + 67\,T^{19} - 82\,T^{20} + 136\,T^{21} -\right.$$

$$\{55, \{\{2, 9\}, \{5, 9\}\}, \text{swapth}, \{2, 5\}\}$$

$$\left(T^{14} + T^{15} - 5\,T^{16} + T^{17} + 7\,T^{18} - 3\,T^{19} - 12\,T^{20} + 30\,T^{21} - 41\,T^{22} + 24\,T^{23} + 19\,T^{24} - 44\,T^{25} + 46\,T^{26} - 86\,T^{27} + 160\,T^2\right.$$

$$\{56, \{\{2, 9\}, \{5, 9\}\}, \text{swapth}, \{2, 9\}\}$$

$$\left(T^{17} + T^{18} - 4\,T^{19} + 6\,T^{21} - 2\,T^{22} - 14\,T^{23} + 31\,T^{24} - 28\,T^{25} + 7\,T^{26} + 14\,T^{27} - 36\,T^{28} + 58\,T^{29} - 83\,T^{30} + 122\,T^{31} - 101\right.$$

$$\{57, \{\{2, 9\}, \{5, 9\}\}, \text{swapth}, \{9, 9\}\}$$

$$\left(T^{17} + T^{18} - 5\,T^{19} + T^{20} + 7\,T^{21} - 3\,T^{22} - 12\,T^{23} + 30\,T^{24} - 41\,T^{25} + 24\,T^{26} + 19\,T^{27} - 44\,T^{28} + 46\,T^{29} - 86\,T^{30} + 160\,T^3\right.$$

$$\{58, \{\{2, 9\}, \{5, 9\}\}, \text{tm}, \{2, 9, 2\}\}$$

$$\left(T^{17} + T^{18} - 5\,T^{19} + T^{20} + 7\,T^{21} - 3\,T^{22} - 12\,T^{23} + 30\,T^{24} - 41\,T^{25} + 24\,T^{26} + 19\,T^{27} - 44\,T^{28} + 46\,T^{29} - 86\,T^{30} + 160\,T^3\right.$$

$$\{59, \{\{2\}, \{5, 9\}\}, \text{swapth}, \{2, 5\}\}$$

$$\left(T^{24} + T^{25} - 5\,T^{26} + T^{27} + 7\,T^{28} - 3\,T^{29} - 12\,T^{30} + 30\,T^{31} - 41\,T^{32} + 24\,T^{33} + 19\,T^{34} - 44\,T^{35} + 46\,T^{36} - 86\,T^{37} + 160\,T^3\right.$$

$$\{60, \{\{2\}, \{5, 9\}\}, \text{swapth}, \{2, 5\}\}$$

$$\left(T^{31} + T^{32} - 5\,T^{33} + T^{34} + 7\,T^{35} - 3\,T^{36} - 12\,T^{37} + 30\,T^{38} - 41\,T^{39} + 24\,T^{40} + 19\,T^{41} - 44\,T^{42} + 46\,T^{43} - 86\,T^{44} + 160\,T^4\right.$$

$$\{61, \{\{2\}, \{5, 9\}\}, \text{swapth}, \{2, 9\}\}$$

$$\left(T^{34} + T^{35} - 5\,T^{36} + T^{37} + 7\,T^{38} - 3\,T^{39} - 12\,T^{40} + 30\,T^{41} - 41\,T^{42} + 24\,T^{43} + 19\,T^{44} - 44\,T^{45} + 46\,T^{46} - 86\,T^{47} + 160\,T^4\right.$$

$$\{62, \{\{2\}, \{5, 9\}\}, \text{swapth}, \{2, 9\}\}$$

$$\left(T^{37} + T^{38} - 5\,T^{39} + T^{40} + 7\,T^{41} - 3\,T^{42} - 12\,T^{43} + 30\,T^{44} - 41\,T^{45} + 24\,T^{46} + 19\,T^{47} - 44\,T^{48} + 46\,T^{49} - 86\,T^{50} + 160\,T^5\right.$$

{63, {{2}, {5, 9}}, swapth, {2, 5}}

$$\left(T^{44}+T^{45}-5\,T^{46}+T^{47}+7\,T^{48}-3\,T^{49}-12\,T^{50}+30\,T^{51}-41\,T^{52}+24\,T^{53}+19\,T^{54}-44\,T^{55}+46\,T^{56}-86\,T^{57}+160\,T^{5}\right.$$

{64, {{2}, {5, 9}}, swapth, {2, 5}}

$$\left(T^{51}+T^{52}-5\,T^{53}+T^{54}+7\,T^{55}-3\,T^{56}-12\,T^{57}+30\,T^{58}-41\,T^{59}+24\,T^{60}+19\,T^{61}-44\,T^{62}+46\,T^{63}-86\,T^{64}+160\,T^{6}\right.$$

{65, {{2}, {5, 9}}, swapth, {2, 5}}

$$\left(T^{58}+T^{59}-5\,T^{60}+T^{61}+7\,T^{62}-3\,T^{63}-12\,T^{64}+30\,T^{65}-41\,T^{66}+24\,T^{67}+19\,T^{68}-44\,T^{69}+46\,T^{70}-86\,T^{71}+160\,T^{7}\right.$$

{66, {{2}, {5, 9}}, swapth, {2, 5}}

$$\left(T^{65}+T^{66}-5\,T^{67}+T^{68}+7\,T^{69}-3\,T^{70}-12\,T^{71}+30\,T^{72}-41\,T^{73}+24\,T^{74}+19\,T^{75}-44\,T^{76}+46\,T^{77}-86\,T^{78}+160\,T^{7}\right.$$

{67, {{2}, {5, 9}}, swapth, {2, 5}}

$$\left(T^{72}+T^{73}-5\,T^{74}+T^{75}+7\,T^{76}-3\,T^{77}-12\,T^{78}+30\,T^{79}-41\,T^{80}+24\,T^{81}+19\,T^{82}-44\,T^{83}+46\,T^{84}-86\,T^{85}+160\,T^{8}\right.$$

{68, {{2}, {5, 9}}, swapth, {2, 5}}

$$\left(T^{79}+T^{80}-5\,T^{81}+T^{82}+7\,T^{83}-3\,T^{84}-12\,T^{85}+30\,T^{86}-41\,T^{87}+24\,T^{88}+19\,T^{89}-44\,T^{90}+46\,T^{91}-86\,T^{92}+160\,T^{9}\right.$$

{69, {{2}, {5, 9}}, swapth, {2, 9}}

$$\left(T^{82}+T^{83}-5\,T^{84}+T^{85}+7\,T^{86}-3\,T^{87}-12\,T^{88}+30\,T^{89}-41\,T^{90}+24\,T^{91}+19\,T^{92}-44\,T^{93}+46\,T^{94}-86\,T^{95}+160\,T^{9}\right.$$

{70, {{2}, {5, 9}}, swapth, {2, 9}}

$$\left(T^{85}+T^{86}-5\,T^{87}+T^{88}+7\,T^{89}-3\,T^{90}-12\,T^{91}+30\,T^{92}-41\,T^{93}+24\,T^{94}+19\,T^{95}-44\,T^{96}+46\,T^{97}-86\,T^{98}+160\,T^{9}\right.$$

{71, {{2}, {5, 9}}, swapth, {2, 5}}

$$\left(T^{92}+T^{93}-5\,T^{94}+T^{95}+7\,T^{96}-3\,T^{97}-12\,T^{98}+30\,T^{99}-41\,T^{100}+24\,T^{101}+19\,T^{102}-44\,T^{103}+46\,T^{104}-86\,T^{105}+1\right.$$

{72, {{2}, {5, 9}}, hm, {9, 5, 9}}

$$\left(T^{92}+T^{93}-5\,T^{94}+T^{95}+7\,T^{96}-3\,T^{97}-12\,T^{98}+30\,T^{99}-41\,T^{100}+24\,T^{101}+19\,T^{102}-44\,T^{103}+46\,T^{104}-86\,T^{105}+1\right.$$

$$\left\{\text{True},\ \left(T^{92}+T^{93}-5\,T^{94}+T^{95}+7\,T^{96}-3\,T^{97}-12\,T^{98}+30\,T^{99}-41\,T^{100}+24\,T^{101}+19\,T^{102}-44\,T^{103}+46\,T\right.\right.$$

$$\text{Det}\left[\begin{pmatrix} \frac{2-4\,T+T^2-3\,T^3+15\,T^4-13\,T^5-9\,T^6+15\,T^7-4\,T^9-4\,T^{10}+6\,T^{11}-2\,T^{12}}{T^7} & \frac{-2+4\,T-T^2+3\,T^3-15\,T^4+13\,T^5+9\,T^6-15\,T^7+4\,T^9+4\,T^{10}-6\,T^{11}+2\,T^{12}}{T^8} \\ T-1 & \frac{1}{T}-1 \end{pmatrix}\right]$$

$$0$$

$$\begin{pmatrix} \frac{-1+2\,T-2\,T^3+T^4+T^5}{T^6} & h[3] & h[5] & h[9] \\ t[1] & \frac{(-1+T)^2\,(1+T)\,(1-T+T^3)}{T^6} & \frac{-1+2\,T-2\,T^3+T^4+T^5-2\,T^7+T^8}{T^6} & \frac{1-T-3\,T^2+5\,T^3-4\,T^5+T^6+T^7}{T^6} \\ t[2] & \frac{(-1+T)^2\,(1+T)\,(1-T+T^3)}{T^7} & -\frac{1-4\,T+5\,T^2-3\,T^4+T^6-T^8+T^9}{T^7} & -\frac{(1-T+T^3)\,(-1+T+2\,T^2-3\,T^3+T^5)}{T^7} \\ t[5] & -\frac{(-1+T)^2\,(1-T+T^3)}{T^7} & \frac{(-1+T)\,(-1+T+T^3)\,(1-T+T^4)}{T^7} & -\frac{(-1+T)^2\,(1-T+T^3)}{T^7} \\ t[6] & 0 & -\frac{1-2\,T+T^3}{T^9} & 0 \\ t[8] & 0 & 0 & \frac{(-1+T)\,(-1+2\,T-2\,T^3+T^4+T^5)}{T^5} \\ t[10] & \frac{(-1+T)\,(1-T+T^3)}{T^6} & \frac{-1+3\,T-2\,T^2+T^3-2\,T^4+T^5}{T^5} & \frac{(-1+T)\,(1-T+T^3)}{T^6} \\ \text{"}1+\Sigma/\omega\text{"} & T & \frac{1}{T^3} & T^2 \end{pmatrix}$$

$$\text{Det}\left[\begin{pmatrix} \frac{-1+3\,T-2\,T^2+T^3-2\,T^4+T^5}{T^5} & \frac{(-1+T)\,(1-T+T^3)}{T^6} \\ \frac{1}{T^3}-1 & T^2-1 \end{pmatrix}\right] \text{// Factor}$$

$$\frac{(-1+T)^2\,\left(1+2\,T^4-2\,T^6-T^7+T^9\right)}{T^9}$$

$$\frac{-1+2\,T-2\,T^3+T^4+T^5}{T^6} \text{// Factor}$$

$$\frac{\left(-1+T+T^2\right)\,\left(1-T+T^3\right)}{T^6}$$

$\beta[14]$

$$\begin{pmatrix} 1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9 & h[1] \\ t[2] & -T^5+3\,T^6-3\,T^7+T^8 \\ t[3] & -T^5+3\,T^6-T^7-5\,T^8+4\,T^9+5\,T^{10}-9\,T^{11}+5\,T^{12}-T^{13} \\ t[4] & -T^3+T^4+T^5-3\,T^6+T^7+5\,T^8-4\,T^9-5\,T^{10}+9\,T^{11}-5\,T^{12}+T^1 \\ t[5] & 0 \\ t[6] & -1+T+T^2-4\,T^3+3\,T^4+6\,T^5-8\,T^6-5\,T^7+14\,T^8-9\,T^9+2\,T^1 \\ t[7] & -T^5+4\,T^6-6\,T^7+4\,T^8-T^9 \\ t[8] & -T^4+2\,T^5-T^6 \\ t[9] & -T+T^2+3\,T^3-4\,T^4-4\,T^5+9\,T^6+T^7-14\,T^8+14\,T^9-6\,T^{10}+T \\ t[10] & -T^4+4\,T^5-6\,T^6+4\,T^7-T^8 \\ 1+\Sigma/\omega & T^2 \end{pmatrix}$$

```
renmin[B[ω_, _, μ_], {i1_, i2_}, {j1_, j2_}] :=
 bbSimplify[1/ω Det[(∂t[i1],h[j1] μ   ∂t[i1],h[j2] μ
                     ∂t[i2],h[j1] μ   ∂t[i2],h[j2] μ)]]
```

```
renmin[β[14], {9, 10}, {1, 2}]
```

$$T^2 - 3\,T^3 - T^4 + 17\,T^5 - 34\,T^6 + 35\,T^7 - 21\,T^8 + 7\,T^9 - T^{10}$$

```
renmin[β[14], {8, 10}, {1, 2}]
```

$$T^5 - 4\,T^6 + 6\,T^7 - 4\,T^8 + T^9$$

```
renmin[β[14], {7, 10}, {1, 2}]
```

$$0$$

```
renmin[β[14], {7, 10}, {1, 3}]
```

$$0$$

```
Collect[β[14][[3]] / β[14][[1]] /. _h → 1, _t, Factor]
```

$$\frac{(-1+T)\,T^2\left(1+T^2+4\,T^3-12\,T^4+8\,T^5+T^6-3\,T^7+T^8\right)t[2]}{1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9} -$$
$$\left((-1+T)\,T^3\left(-4+2\,T+3\,T^2+2\,T^3-4\,T^4-6\,T^5+8\,T^6-3\,T^8+T^9\right)t[3]\right)/$$
$$\left(1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9\right) + (-1+T)\,T^3\,t[4] + (-1+T)\,T^2\,t[5] +$$
$$\left((-1+T)\left(1+3\,T+T^3-4\,T^4-6\,T^5+8\,T^6+5\,T^7-9\,T^8+3\,T^9\right)t[6]\right)/$$
$$\left(1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9\right) +$$
$$\frac{(-1+T)\,T^2\left(2-T+5\,T^3-15\,T^4+16\,T^5-7\,T^6+T^7\right)t[7]}{1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9} +$$
$$\frac{(-1+T)\,T^3\left(2-2\,T+T^2\right)\left(1+3\,T-2\,T^2-2\,T^3+T^4\right)t[8]}{1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9} +$$
$$\left((-1+T)\left(6-2\,T-10\,T^2+4\,T^3+10\,T^4-2\,T^5-13\,T^6+8\,T^7+3\,T^8-4\,T^9+T^{10}\right)t[9]\right)/$$
$$\left(1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9\right) +$$
$$\frac{(-1+T)\,T\left(2-T+5\,T^3-15\,T^4+16\,T^5-7\,T^6+T^7\right)t[10]}{1-T^2+2\,T^3+T^4-4\,T^5+5\,T^7-4\,T^8+T^9}$$