Pensieve header: A Θ relation?

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In[1]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.1"];
<< betaCalculus.m
Clear[ℏ]; Unprotect[C];
$PerturbativeDegree = 4;
βSimplify[expr_] := Replace[
   Series[Normal[expr], {ℏ, 0, $PerturbativeDegree}],
   sd_SeriesData :→ MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[βSimplify[ω], βSimplify[μ]];
{V, C, sol} = Get[Switch[$PerturbativeDegree,
    4, "SolutionToDegree4-120523.m",
    6, "SolutionToDegree6-120523.m",
    8, "SolutionToDegree8-120524.m"
   ]];
C = C /. κ1 → 0;
Φ = (Inverse[V] // dP[12, 3]) ** Inverse[V] ** (V // dP[2, 3]) ** (V // dP[1, 23]);
ν = B[Series[Sinh[c1 ℏ / 2] / (c1 ℏ / 2), {ℏ, 0, $PerturbativeDegree}], 0];
```

```
In[11]:= {
   "R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
   "TwistEq" → V ** Θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
   "Unitarity" → V ** (V // dA[1] // dA[2]) == B[1, 0],
   "VerticalFlipForV" → V ** (V // dS[1] // dS[2]) == R[1, 2],
   "CapEquation" → (V ** (C // dP[12]) // dcap[1] // dcap[2]) ==
     (C * (C // dP[2]) // dcap[1] // dcap[2]),
   "VSidesDelete" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
   "CapsAndCups" → C == (C // dS[1]) /. κ1 → 0,
   "Pentagon" → Φ ** (Φ // dP[1, 23, 4]) ** (Φ // dP[2, 3, 4]) ==
     (Φ // dP[12, 3, 4]) ** (Φ // dP[1, 2, 34]),
   "PositiveHexagon" → (Θ[1, 2, +1] // dP[12, 3]) ==
     (Φ ** Θ[2, 3, +1] ** Inverse[Φ // dP[1, 3, 2]] ** Θ[1, 3, +1] ** (Φ // dP[3, 1, 2])),
   "NegativeHexagon" → (Θ[1, 2, -1] // dP[12, 3]) ==
     (Φ ** Θ[2, 3, -1] ** Inverse[Φ // dP[1, 3, 2]] ** Θ[1, 3, -1] ** (Φ // dP[3, 1, 2])),
   "HorizontalFlipForΦ" → Φ ** (Φ // dP[3, 2, 1]) == B[1, 0],
   "VerticalFlipForΦ" → Φ ** (Φ // dS[1] // dS[2] // dS[3]) == B[1, 0],
   "OverhandEquation" →
    (Φ // dΔ[1, 0, 1] // dS[2] // dS[3] // dm[0, 3, 0] // dm[1, 2, 1]) == B[1, 0],
   "ValueOfν" → (Φ // dS[2] // dm[3, 2, 2] // dm[2, 1, 1]) == ν,
   "ValueOfC" → Inverse[C ** C ** C ** C] == ν,
   "VTopDelete" →
    (V // dS[2] // dm[1, 2, 1]) == Inverse[C ** C] ** (R[1, 1, -1 / 2] // dS[1]),
   "EKTopCapLeftPuncture" →
    (V // tη[1] // dm[2, 3, 2] // dS[2] // hm[1, 2, 1]) == B[1, 0],
   "EKRightCupLeftPuncture" →
    (V // dm[3, 2, 2] // hη[2] // tη[1] // dm[1, 2, 1]) == B[1, 0],
   "EKRightCupTopPuncture" →
    (V // dm[3, 2, 2] // hη[2] // dS[1] // dm[2, 1, 1]) == Inverse[C ** C],
   "EKTopCapRightPuncture" →
    (V // tη[2] // dm[1, 3, 1] // dS[1] // dm[2, 1, 1]) == R[1, 1, -1 / 2],
   "EKLeftCupRightPuncture" →
    (V // dm[3, 1, 1] // hη[1] // tη[2] // dm[2, 1, 1]) == R[1, 1, 1 / 2],
   "EKLeftCupTopPuncture" → (V // dm[3, 1, 1] // hη[1] // dS[2] // dm[1, 2, 1]) ==
     Inverse[(R[1, 1, 1 / 2] // dS[1]) ** C ** C]
  } // ColumnForm
```

Out[11]=
```
R4 → True
TwistEq → True
Unitarity → True
VerticalFlipForV → True
CapEquation → True
VSidesDelete → True
CapsAndCups → True
Pentagon → True
PositiveHexagon → True
NegativeHexagon → True
HorizontalFlipForΦ → True
VerticalFlipForΦ → True
OverhandEquation → True
ValueOfν → True
ValueOfC → True
VTopDelete → True
EKTopCapLeftPuncture → True
EKRightCupLeftPuncture → True
EKRightCupTopPuncture → True
EKTopCapRightPuncture → True
EKLeftCupRightPuncture → True
EKLeftCupTopPuncture → True
```

In[16]:= **?? Θ**

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Global`Θ
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$$\Theta[x\_, y\_, p\_] := d\triangle[x, x, y]\left[R\left[x, x, \frac{p}{2}\right]\right] ** R\left[x, x, -\frac{p}{2}\right] ** R\left[y, y, -\frac{p}{2}\right]$$

$$\Theta[x\_, y\_] := \Theta[x, y, 1]$$

In[15]:= **Θ[1, 2, 2]**

Out[15]=

$$\begin{pmatrix} 1 & h[1] \\ t[1] & -\frac{c_2 \hbar}{2} + \left(-\frac{1}{6} c_1 c_2 - \frac{c_2^2}{3}\right)\hbar^2 + \left(-\frac{1}{24} c_1^2 c_2 - \frac{1}{8} c_1 c_2^2 - \frac{c_2^3}{8}\right)\hbar^3 + \left(-\frac{1}{120} c_1^3 c_2 - \frac{1}{30} c_1^2 c_2^2 - \frac{1}{20} c_1 c_2^3 \right. \\ t[2] & 1 + \left(\frac{c_1}{2} + \frac{c_2}{2}\right)\hbar + \left(\frac{c_1^2}{6} + \frac{c_1 c_2}{3} + \frac{c_2^2}{6}\right)\hbar^2 + \left(\frac{c_1^3}{24} + \frac{1}{8} c_1^2 c_2 + \frac{1}{8} c_1 c_2^2 + \frac{c_2^3}{24}\right)\hbar^3 + \left(\frac{c_1^4}{120} + \frac{1}{30} c_1^3 c_2 + \frac{1}{20} c_1^2 c_2^2 + \right. \end{pmatrix}$$

In[13]:= **R[1, 2] ** R[2, 1] ** R[1, 1, -1] ** R[2, 2, -1]**

Out[13]=

$$\begin{pmatrix} 1 & h[1] \\ t[1] & -1 + \left(\frac{c_1}{2} - c_2\right)\hbar + \left(-\frac{c_1^2}{6} + \frac{c_1 c_2}{2} - \frac{c_2^2}{2}\right)\hbar^2 + \left(\frac{c_1^3}{24} - \frac{1}{6} c_1^2 c_2 + \frac{1}{4} c_1 c_2^2 - \frac{c_2^3}{6}\right)\hbar^3 + \left(-\frac{c_1^4}{120} + \frac{1}{24} c_1^3 c_2 - \frac{1}{12} c_1^2 c \right. \\ t[2] & 1 + \frac{c_2 \hbar}{2} + \frac{1}{6} c_2^2 \hbar^2 + \frac{1}{24} c_2^3 \hbar^3 + \frac{1}{120} c_2^4 \hbar^4 + O[\hbar]^5 \end{pmatrix}$$