Pensieve header: Perturbative β-calculations.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.1"];
<< betaCalculus.m
```

```
Clear[ħ];
$PerturbativeDegree = 8;
βSimplify[expr_] := Replace[
   Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
   sd_SeriesData :> MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[
   βSimplify[ω],
   βSimplify[μ]
  ];
Unprotect[C];
{V, C, sol6} = Get["SolutionToDegree6-120523.m"];
```

The Knot-Theoretic Equations

```
{
  V0 = βCollect[
       B[ω[ħ c1, ħ c2], α[ħ c1, ħ c2] t[1] h[1] +
         β[ħ c1, ħ c2] t[1] h[2] + γ[ħ c1, ħ c2] t[2] h[1] + δ[ħ c1, ħ c2] t[2] h[2]]
      ] /. {
       (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0,
       (ϵ : (α | β | γ | δ | ω | κ))^(k___)[___] :> ϵFromDigits[{k}]
      } /. sol6,
  C0 = βCollect[B[κ[ħ c1], 0]] /. {
       (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0, (ϵ : (α | β | γ | δ | ω | κ))^(k___)[___] :> ϵFromDigits[{k}]
      } /. sol6,
  eqns0 = GroupLikeQ[V0],
  eqns1 = HardR4[V0],
  eqns2 = TwistEq[V0],
  eqns3 = And[(V0 // dη[1]) == B[1, 0], (V0 // dη[2]) == B[1, 0]],
  eqns4 = V0 ** (V0 // dA[1] // dA[2]) == B[1, 0],
  eqns5 = CapEquation[V0, C0],
  eqns6 = (C0 // tη[1]) == B[1, 0],
  eqns7 = (V0 == Rot120[V0]),
  eqns8 = V0 ** (V0 // dS[1] // dS[2]) == R[1, 2]
 } // ColumnForm
```

A very large output was generated. Here is a sample of it:

<<1>>

Show Less | Show More | Show Full Output | Set Size Limit...

```
eqns = (eqns0 && eqns1 && eqns2 && eqns3 && eqns4 && eqns5 && eqns6 && eqns8);
```

```
Timing[sol8 = PerturbativeSolveAlways[eqns, ħ, $PerturbativeDegree, {c1, c2}]]
```

$$\left\{0.484,\ \left\{\alpha_{70}\to 0,\ \beta_{70}\to \frac{1}{2048},\ \gamma_7\to 0,\ \alpha_7\to -\frac{1}{480},\ \alpha_{16}\to -\frac{19}{13\,440},\ \alpha_{25}\to -\frac{271}{241\,920},\right.\right.$$
$$\alpha_{34}\to -\frac{2893}{3\,225\,600},\ \alpha_{43}\to -\frac{2399}{3\,225\,600},\ \alpha_{52}\to -\frac{127}{215\,040},\ \alpha_{61}\to -\frac{127}{215\,040},\ \delta_7\to 0,\ \beta_7\to -\frac{1}{240},$$
$$\beta_{16}\to -\frac{1}{560},\ \beta_{25}\to -\frac{53}{60\,480},\ \beta_{34}\to -\frac{167}{201\,600},\ \beta_{43}\to -\frac{2347}{3\,225\,600},\ \beta_{52}\to -\frac{97}{215\,040},$$
$$\beta_{61}\to -\frac{11}{215\,040},\ \alpha_{80}\to 0,\ \beta_{80}\to \frac{1}{4608},\ \gamma_8\to 0,\ \alpha_8\to 0,\ \alpha_{17}\to 0,\ \alpha_{26}\to 0,\ \alpha_{35}\to 0,\ \alpha_{44}\to 0,$$
$$\alpha_{53}\to 0,\ \alpha_{62}\to 0,\ \alpha_{71}\to 0,\ \delta_8\to 0,\ \gamma_{70}\to \frac{127}{30\,720},\ \beta_8\to 0,\ \gamma_{61}\to \frac{127}{71\,680},\ \beta_{17}\to -\frac{1}{960},$$
$$\gamma_{52}\to \frac{2399}{1\,935\,360},\ \beta_{26}\to -\frac{1}{2688},\ \gamma_{43}\to \frac{2893}{3\,225\,600},\ \beta_{35}\to -\frac{1}{7680},\ \gamma_{34}\to \frac{271}{403\,200},$$
$$\beta_{44}\to -\frac{37}{230\,400},\ \gamma_{25}\to \frac{19}{40\,320},\ \beta_{53}\to -\frac{7}{46\,080},\ \gamma_{16}\to \frac{1}{3360},\ \beta_{62}\to -\frac{1}{15\,360},$$
$$\beta_{71}\to \frac{1}{9216},\ \omega_7\to 0,\ \omega_{16}\to 0,\ \omega_{25}\to 0,\ \omega_{34}\to 0,\ \omega_{43}\to 0,\ \omega_{52}\to 0,\ \omega_{61}\to 0,\ \omega_{70}\to 0,$$
$$\kappa_7\to \frac{\kappa_1\,(-153+1456\,\kappa_1^2-5376\,\kappa_1^4+12\,288\,\kappa_1^6)}{12\,288},\ \omega_{44}\to \frac{20\,509}{12\,902\,400},\ \gamma_{26}\to 0,\ \gamma_{35}\to 0,$$
$$\gamma_{44}\to 0,\ \gamma_{53}\to 0,\ \gamma_{62}\to 0,\ \gamma_{71}\to 0,\ \gamma_{80}\to 0,\ \delta_{17}\to 0,\ \delta_{26}\to \frac{1}{3360},\ \delta_{35}\to \frac{1}{5376},\ \delta_{44}\to \frac{1}{9600},$$
$$\delta_{53}\to \frac{37}{184\,320},\ \delta_{71}\to \frac{7}{30\,720},\ \omega_8\to 0,\ \omega_{17}\to \frac{1}{960},\ \omega_{26}\to \frac{37}{26\,880},\ \omega_{35}\to \frac{991}{645\,120},$$
$$\omega_{53}\to \frac{991}{645\,120},\ \omega_{62}\to \frac{37}{26\,880},\ \omega_{71}\to \frac{1}{960},\ \omega_{80}\to 0,\ \delta_{62}\to \frac{7}{23\,040},\ \gamma_{17}\to 0,\ \delta_{80}\to -\frac{1}{1152},$$
$$\kappa_8\to \frac{1}{589\,824}\left(1169-29\,376\,\kappa_1^2+139\,776\,\kappa_1^4-344\,064\,\kappa_1^6+589\,824\,\kappa_1^8\right),\ \delta_{43}\to \frac{167}{201\,600},$$
$$\left.\left.\delta_{25}\to \frac{1}{1680},\ \delta_{52}\to \frac{2347}{1\,935\,360},\ \delta_{61}\to \frac{97}{71\,680},\ \delta_{16}\to \frac{1}{1680},\ \delta_{34}\to \frac{53}{100\,800},\ \delta_{70}\to \frac{11}{30\,720}\right\}\right\}$$

```
{V0, C0} /. sol8
```

$$\left\{\begin{array}{l} 1-\frac{1}{48}\,(c_1\,c_2)\,\hbar^2+\left(\frac{c_1^3\,c_2}{2880}+\frac{17\,c_1^2\,c_2^2}{23\,040}+\frac{c_1\,c_2^3}{2880}\right)\hbar^4+\left(-\frac{c_1^5\,c_2}{120\,960}-\frac{c_1^4\,c_2^2}{35\,840}-\frac{103\,c_1^3\,c_2^3}{2\,580\,480}-\frac{c_1^2\,c_2^4}{35\,840}-\frac{c_1\,c_2^5}{120\,960}\right)\hbar^6+\left(\frac{c_1^7\,c_2}{4\,838\,400}\right. \\ t[1] \\ t[2] \end{array}\right.$$

```
sol = Union[sol6, sol8]
```

$$\left\{\alpha_0\to 0,\ \alpha_1\to \frac{1}{24},\ \alpha_2\to 0,\ \alpha_3\to -\frac{1}{240},\ \alpha_4\to 0,\ \alpha_5\to \frac{1}{504},\ \alpha_6\to 0,\ \alpha_7\to -\frac{1}{480},\ \alpha_8\to 0,\right.$$
$$\alpha_{10}\to 0,\ \alpha_{11}\to 0,\ \alpha_{12}\to -\frac{7}{2880},\ \alpha_{13}\to 0,\ \alpha_{14}\to \frac{13}{10\,080},\ \alpha_{15}\to 0,\ \alpha_{16}\to -\frac{19}{13\,440},\ \alpha_{17}\to 0,$$
$$\alpha_{20}\to 0,\ \alpha_{21}\to -\frac{7}{2880},\ \alpha_{22}\to 0,\ \alpha_{23}\to \frac{83}{80\,640},\ \alpha_{24}\to 0,\ \alpha_{25}\to -\frac{271}{241\,920},\ \alpha_{26}\to 0,\ \alpha_{30}\to 0,$$
$$\alpha_{31}\to 0,\ \alpha_{32}\to \frac{31}{40\,320},\ \alpha_{33}\to 0,\ \alpha_{34}\to -\frac{2893}{3\,225\,600},\ \alpha_{35}\to 0,\ \alpha_{40}\to 0,\ \alpha_{41}\to \frac{31}{40\,320},$$

$$\alpha_{42} \to 0,\ \alpha_{43} \to -\frac{2399}{3\,225\,600},\ \alpha_{44} \to 0,\ \alpha_{50} \to 0,\ \alpha_{51} \to 0,\ \alpha_{52} \to -\frac{127}{215\,040},\ \alpha_{53} \to 0,\ \alpha_{60} \to 0,$$

$$\alpha_{61} \to -\frac{127}{215\,040},\ \alpha_{62} \to 0,\ \alpha_{70} \to 0,\ \alpha_{71} \to 0,\ \alpha_{80} \to 0,\ \beta_0 \to \frac{1}{2},\ \beta_1 \to \frac{1}{12},\ \beta_2 \to 0,\ \beta_3 \to -\frac{1}{120},$$

$$\beta_4 \to 0,\ \beta_5 \to \frac{1}{252},\ \beta_6 \to 0,\ \beta_7 \to -\frac{1}{240},\ \beta_8 \to 0,\ \beta_{10} \to \frac{1}{8},\ \beta_{11} \to \frac{1}{48},\ \beta_{12} \to -\frac{1}{360},\ \beta_{13} \to -\frac{1}{480},$$

$$\beta_{14} \to \frac{1}{630},\ \beta_{15} \to \frac{1}{1008},\ \beta_{16} \to -\frac{1}{560},\ \beta_{17} \to -\frac{1}{960},\ \beta_{20} \to \frac{1}{24},\ \beta_{21} \to \frac{19}{2880},\ \beta_{22} \to -\frac{1}{2880},$$

$$\beta_{23} \to \frac{11}{40\,320},\ \beta_{24} \to \frac{1}{3360},\ \beta_{25} \to -\frac{53}{60\,480},\ \beta_{26} \to -\frac{1}{2688},\ \beta_{30} \to \frac{1}{64},\ \beta_{31} \to \frac{1}{320},\ \beta_{32} \to \frac{53}{80\,640},$$

$$\beta_{33} \to -\frac{1}{10\,752},\ \beta_{34} \to -\frac{167}{201\,600},\ \beta_{35} \to -\frac{1}{7680},\ \beta_{40} \to \frac{1}{160},\ \beta_{41} \to \frac{17}{10\,080},\ \beta_{42} \to \frac{1}{8064},$$

$$\beta_{43} \to -\frac{2347}{3\,225\,600},\ \beta_{44} \to -\frac{37}{230\,400},\ \beta_{50} \to \frac{1}{384},\ \beta_{51} \to \frac{1}{1792},\ \beta_{52} \to -\frac{97}{215\,040},\ \beta_{53} \to -\frac{7}{46\,080},$$

$$\beta_{60} \to \frac{1}{896},\ \beta_{61} \to -\frac{11}{215\,040},\ \beta_{62} \to -\frac{1}{15\,360},\ \beta_{70} \to \frac{1}{2048},\ \beta_{71} \to \frac{1}{9216},\ \beta_{80} \to \frac{1}{4608},\ \gamma_0 \to 0,$$

$$\gamma_1 \to 0,\ \gamma_2 \to 0,\ \gamma_3 \to 0,\ \gamma_4 \to 0,\ \gamma_5 \to 0,\ \gamma_6 \to 0,\ \gamma_7 \to 0,\ \gamma_8 \to 0,\ \gamma_{10} \to -\frac{1}{24},\ \gamma_{11} \to 0,$$

$$\gamma_{12} \to \frac{1}{720},\ \gamma_{13} \to 0,\ \gamma_{14} \to -\frac{1}{2520},\ \gamma_{15} \to 0,\ \gamma_{16} \to \frac{1}{3360},\ \gamma_{17} \to 0,\ \gamma_{20} \to 0,\ \gamma_{21} \to \frac{7}{2880},$$

$$\gamma_{22} \to 0,\ \gamma_{23} \to -\frac{13}{20\,160},\ \gamma_{24} \to 0,\ \gamma_{25} \to \frac{19}{40\,320},\ \gamma_{26} \to 0,\ \gamma_{30} \to \frac{7}{960},\ \gamma_{31} \to 0,\ \gamma_{32} \to -\frac{83}{80\,640},$$

$$\gamma_{33} \to 0,\ \gamma_{34} \to \frac{271}{403\,200},\ \gamma_{35} \to 0,\ \gamma_{40} \to 0,\ \gamma_{41} \to -\frac{31}{20\,160},\ \gamma_{42} \to 0,\ \gamma_{43} \to \frac{2893}{3\,225\,600},$$

$$\gamma_{44} \to 0,\ \gamma_{50} \to -\frac{31}{8064},\ \gamma_{51} \to 0,\ \gamma_{52} \to \frac{2399}{1\,935\,360},\ \gamma_{53} \to 0,\ \gamma_{60} \to 0,\ \gamma_{61} \to \frac{127}{71\,680},\ \gamma_{62} \to 0,$$

$$\gamma_{70} \to \frac{127}{30\,720},\ \gamma_{71} \to 0,\ \gamma_{80} \to 0,\ \delta_0 \to 0,\ \delta_1 \to 0,\ \delta_2 \to 0,\ \delta_3 \to 0,\ \delta_4 \to 0,\ \delta_5 \to 0,\ \delta_6 \to 0,$$

$$\delta_7 \to 0,\ \delta_8 \to 0,\ \delta_{10} \to -\frac{1}{12},\ \delta_{11} \to 0,\ \delta_{12} \to \frac{1}{360},\ \delta_{13} \to 0,\ \delta_{14} \to -\frac{1}{1260},\ \delta_{15} \to 0,\ \delta_{16} \to \frac{1}{1680},$$

$$\delta_{17} \to 0,\ \delta_{20} \to -\frac{1}{24},\ \delta_{21} \to \frac{1}{360},\ \delta_{22} \to \frac{1}{720},\ \delta_{23} \to -\frac{1}{1260},\ \delta_{24} \to -\frac{1}{2520},\ \delta_{25} \to \frac{1}{1680},$$

$$\delta_{26} \to \frac{1}{3360},\ \delta_{30} \to -\frac{19}{960},\ \delta_{31} \to \frac{1}{1920},\ \delta_{32} \to -\frac{11}{40\,320},\ \delta_{33} \to -\frac{1}{4480},\ \delta_{34} \to \frac{53}{100\,800},$$

$$\delta_{35} \to \frac{1}{5376},\ \delta_{40} \to -\frac{1}{80},\ \delta_{41} \to -\frac{53}{40\,320},\ \delta_{42} \to \frac{1}{8064},\ \delta_{43} \to \frac{167}{201\,600},\ \delta_{44} \to \frac{1}{9600},$$

$$\delta_{50} \to -\frac{17}{2016},\ \delta_{51} \to -\frac{5}{16\,128},\ \delta_{52} \to \frac{2347}{1\,935\,360},\ \delta_{53} \to \frac{37}{184\,320},\ \delta_{60} \to -\frac{3}{896},\ \delta_{61} \to \frac{97}{71\,680},$$

$$\delta_{62} \to \frac{7}{23\,040},\ \delta_{70} \to \frac{11}{30\,720},\ \delta_{71} \to \frac{7}{30\,720},\ \delta_{80} \to -\frac{1}{1152},\ \kappa_0 \to 1,\ \kappa_2 \to \frac{1}{48}\left(-1 + 48\,\kappa_1^2\right),$$

$$\kappa_3 \to \frac{1}{16}\,\kappa_1\left(-1 + 16\,\kappa_1^2\right),\ \kappa_4 \to \frac{13 - 480\,\kappa_1^2 + 3840\,\kappa_1^4}{3840},\ \kappa_5 \to \frac{1}{768}\,\kappa_1\left(13 - 160\,\kappa_1^2 + 768\,\kappa_1^4\right),$$

$$\kappa_6 \to \frac{-51 + 1456\,\kappa_1^2 - 8960\,\kappa_1^4 + 28\,672\,\kappa_1^6}{28\,672},\ \kappa_7 \to \frac{\kappa_1\left(-153 + 1456\,\kappa_1^2 - 5376\,\kappa_1^4 + 12\,288\,\kappa_1^6\right)}{12\,288},$$

$$\kappa_8 \to \frac{1}{589\,824}\left(1169 - 29\,376\,\kappa_1^2 + 139\,776\,\kappa_1^4 - 344\,064\,\kappa_1^6 + 589\,824\,\kappa_1^8\right),\ \omega_0 \to 1,\ \omega_1 \to 0,\ \omega_2 \to 0,$$

$$\omega_3 \to 0,\ \omega_4 \to 0,\ \omega_5 \to 0,\ \omega_6 \to 0,\ \omega_7 \to 0,\ \omega_8 \to 0,\ \omega_{10} \to 0,\ \omega_{11} \to -\frac{1}{48},\ \omega_{12} \to 0,\ \omega_{13} \to \frac{1}{480},$$
$$\omega_{14} \to 0,\ \omega_{15} \to -\frac{1}{1008},\ \omega_{16} \to 0,\ \omega_{17} \to \frac{1}{960},\ \omega_{20} \to 0,\ \omega_{21} \to 0,\ \omega_{22} \to \frac{17}{5760},\ \omega_{23} \to 0,$$
$$\omega_{24} \to -\frac{3}{2240},\ \omega_{25} \to 0,\ \omega_{26} \to \frac{37}{26\,880},\ \omega_{30} \to 0,\ \omega_{31} \to \frac{1}{480},\ \omega_{32} \to 0,\ \omega_{33} \to -\frac{103}{71\,680},\ \omega_{34} \to 0,$$
$$\omega_{35} \to \frac{991}{645\,120},\ \omega_{40} \to 0,\ \omega_{41} \to 0,\ \omega_{42} \to -\frac{3}{2240},\ \omega_{43} \to 0,\ \omega_{44} \to \frac{20\,509}{12\,902\,400},\ \omega_{50} \to 0,\ \omega_{51} \to -\frac{1}{1008},$$
$$\omega_{52} \to 0,\ \omega_{53} \to \frac{991}{645\,120},\ \omega_{60} \to 0,\ \omega_{61} \to 0,\ \omega_{62} \to \frac{37}{26\,880},\ \omega_{70} \to 0,\ \omega_{71} \to \frac{1}{960},\ \omega_{80} \to 0\Big\}$$

```
indvars = Union[Flatten[Union[Cases[Last /@ #, ϵ_k_ :> ϵ_k, Infinity]] & /@ {sol}]]
```

$$\{\kappa_1\}$$

```
sol1 = Union[
  sol /. Thread[indvars → 0],
  Thread[indvars → 0]
 ]
```

$$\Big\{\alpha_0 \to 0,\ \alpha_1 \to \frac{1}{24},\ \alpha_2 \to 0,\ \alpha_3 \to -\frac{1}{240},\ \alpha_4 \to 0,\ \alpha_5 \to \frac{1}{504},\ \alpha_6 \to 0,\ \alpha_7 \to -\frac{1}{480},\ \alpha_8 \to 0,$$
$$\alpha_{10} \to 0,\ \alpha_{11} \to 0,\ \alpha_{12} \to -\frac{7}{2880},\ \alpha_{13} \to 0,\ \alpha_{14} \to \frac{13}{10\,080},\ \alpha_{15} \to 0,\ \alpha_{16} \to -\frac{19}{13\,440},\ \alpha_{17} \to 0,$$
$$\alpha_{20} \to 0,\ \alpha_{21} \to -\frac{7}{2880},\ \alpha_{22} \to 0,\ \alpha_{23} \to \frac{83}{80\,640},\ \alpha_{24} \to 0,\ \alpha_{25} \to -\frac{271}{241\,920},\ \alpha_{26} \to 0,\ \alpha_{30} \to 0,$$
$$\alpha_{31} \to 0,\ \alpha_{32} \to \frac{31}{40\,320},\ \alpha_{33} \to 0,\ \alpha_{34} \to -\frac{2893}{3\,225\,600},\ \alpha_{35} \to 0,\ \alpha_{40} \to 0,\ \alpha_{41} \to \frac{31}{40\,320},$$
$$\alpha_{42} \to 0,\ \alpha_{43} \to -\frac{2399}{3\,225\,600},\ \alpha_{44} \to 0,\ \alpha_{50} \to 0,\ \alpha_{51} \to 0,\ \alpha_{52} \to -\frac{127}{215\,040},\ \alpha_{53} \to 0,\ \alpha_{60} \to 0,$$
$$\alpha_{61} \to -\frac{127}{215\,040},\ \alpha_{62} \to 0,\ \alpha_{70} \to 0,\ \alpha_{71} \to 0,\ \alpha_{80} \to 0,\ \beta_0 \to \frac{1}{2},\ \beta_1 \to \frac{1}{12},\ \beta_2 \to 0,\ \beta_3 \to -\frac{1}{120},$$
$$\beta_4 \to 0,\ \beta_5 \to \frac{1}{252},\ \beta_6 \to 0,\ \beta_7 \to -\frac{1}{240},\ \beta_8 \to 0,\ \beta_{10} \to \frac{1}{8},\ \beta_{11} \to \frac{1}{48},\ \beta_{12} \to -\frac{1}{360},\ \beta_{13} \to -\frac{1}{480},$$
$$\beta_{14} \to \frac{1}{630},\ \beta_{15} \to \frac{1}{1008},\ \beta_{16} \to -\frac{1}{560},\ \beta_{17} \to -\frac{1}{960},\ \beta_{20} \to \frac{1}{24},\ \beta_{21} \to \frac{19}{2880},\ \beta_{22} \to -\frac{1}{2880},$$
$$\beta_{23} \to \frac{11}{40\,320},\ \beta_{24} \to \frac{1}{3360},\ \beta_{25} \to -\frac{53}{60\,480},\ \beta_{26} \to -\frac{1}{2688},\ \beta_{30} \to \frac{1}{64},\ \beta_{31} \to \frac{1}{320},\ \beta_{32} \to \frac{53}{80\,640},$$
$$\beta_{33} \to -\frac{1}{10\,752},\ \beta_{34} \to -\frac{167}{201\,600},\ \beta_{35} \to -\frac{1}{7680},\ \beta_{40} \to \frac{1}{160},\ \beta_{41} \to \frac{17}{10\,080},\ \beta_{42} \to \frac{1}{8064},$$
$$\beta_{43} \to -\frac{2347}{3\,225\,600},\ \beta_{44} \to -\frac{37}{230\,400},\ \beta_{50} \to \frac{1}{384},\ \beta_{51} \to \frac{1}{1792},\ \beta_{52} \to -\frac{97}{215\,040},\ \beta_{53} \to -\frac{7}{46\,080},$$
$$\beta_{60} \to \frac{1}{896},\ \beta_{61} \to -\frac{11}{215\,040},\ \beta_{62} \to -\frac{1}{15\,360},\ \beta_{70} \to \frac{1}{2048},\ \beta_{71} \to \frac{1}{9216},\ \beta_{80} \to \frac{1}{4608},\ \gamma_0 \to 0,$$
$$\gamma_1 \to 0,\ \gamma_2 \to 0,\ \gamma_3 \to 0,\ \gamma_4 \to 0,\ \gamma_5 \to 0,\ \gamma_6 \to 0,\ \gamma_7 \to 0,\ \gamma_8 \to 0,\ \gamma_{10} \to -\frac{1}{24},\ \gamma_{11} \to 0,$$
$$\gamma_{12} \to \frac{1}{720},\ \gamma_{13} \to 0,\ \gamma_{14} \to -\frac{1}{2520},\ \gamma_{15} \to 0,\ \gamma_{16} \to \frac{1}{3360},\ \gamma_{17} \to 0,\ \gamma_{20} \to 0,\ \gamma_{21} \to \frac{7}{2880},$$

$\gamma_{22} \to 0,\ \gamma_{23} \to -\frac{13}{20160},\ \gamma_{24} \to 0,\ \gamma_{25} \to \frac{19}{40320},\ \gamma_{26} \to 0,\ \gamma_{30} \to \frac{7}{960},\ \gamma_{31} \to 0,\ \gamma_{32} \to -\frac{83}{80640},$

$\gamma_{33} \to 0,\ \gamma_{34} \to \frac{271}{403200},\ \gamma_{35} \to 0,\ \gamma_{40} \to 0,\ \gamma_{41} \to -\frac{31}{20160},\ \gamma_{42} \to 0,\ \gamma_{43} \to \frac{2893}{3225600},$

$\gamma_{44} \to 0,\ \gamma_{50} \to -\frac{31}{8064},\ \gamma_{51} \to 0,\ \gamma_{52} \to \frac{2399}{1935360},\ \gamma_{53} \to 0,\ \gamma_{60} \to 0,\ \gamma_{61} \to \frac{127}{71680},\ \gamma_{62} \to 0,$

$\gamma_{70} \to \frac{127}{30720},\ \gamma_{71} \to 0,\ \gamma_{80} \to 0,\ \delta_0 \to 0,\ \delta_1 \to 0,\ \delta_2 \to 0,\ \delta_3 \to 0,\ \delta_4 \to 0,\ \delta_5 \to 0,\ \delta_6 \to 0,$

$\delta_7 \to 0,\ \delta_8 \to 0,\ \delta_{10} \to -\frac{1}{12},\ \delta_{11} \to 0,\ \delta_{12} \to \frac{1}{360},\ \delta_{13} \to 0,\ \delta_{14} \to -\frac{1}{1260},\ \delta_{15} \to 0,\ \delta_{16} \to \frac{1}{1680},$

$\delta_{17} \to 0,\ \delta_{20} \to -\frac{1}{24},\ \delta_{21} \to \frac{1}{360},\ \delta_{22} \to \frac{1}{720},\ \delta_{23} \to -\frac{1}{1260},\ \delta_{24} \to -\frac{1}{2520},\ \delta_{25} \to \frac{1}{1680},$

$\delta_{26} \to \frac{1}{3360},\ \delta_{30} \to -\frac{19}{960},\ \delta_{31} \to \frac{1}{1920},\ \delta_{32} \to -\frac{11}{40320},\ \delta_{33} \to -\frac{1}{4480},\ \delta_{34} \to \frac{53}{100800},$

$\delta_{35} \to \frac{1}{5376},\ \delta_{40} \to -\frac{1}{80},\ \delta_{41} \to -\frac{53}{40320},\ \delta_{42} \to \frac{1}{8064},\ \delta_{43} \to \frac{167}{201600},\ \delta_{44} \to \frac{1}{9600},$

$\delta_{50} \to -\frac{17}{2016},\ \delta_{51} \to -\frac{5}{16128},\ \delta_{52} \to \frac{2347}{1935360},\ \delta_{53} \to \frac{37}{184320},\ \delta_{60} \to -\frac{3}{896},\ \delta_{61} \to \frac{97}{71680},$

$\delta_{62} \to \frac{7}{23040},\ \delta_{70} \to \frac{11}{30720},\ \delta_{71} \to \frac{7}{30720},\ \delta_{80} \to -\frac{1}{1152},\ \kappa_0 \to 1,\ \kappa_1 \to 0,\ \kappa_2 \to -\frac{1}{48},\ \kappa_3 \to 0,$

$\kappa_4 \to \frac{13}{3840},\ \kappa_5 \to 0,\ \kappa_6 \to -\frac{51}{28672},\ \kappa_7 \to 0,\ \kappa_8 \to \frac{1169}{589824},\ \omega_0 \to 1,\ \omega_1 \to 0,\ \omega_2 \to 0,\ \omega_3 \to 0,$

$\omega_4 \to 0,\ \omega_5 \to 0,\ \omega_6 \to 0,\ \omega_7 \to 0,\ \omega_8 \to 0,\ \omega_{10} \to 0,\ \omega_{11} \to -\frac{1}{48},\ \omega_{12} \to 0,\ \omega_{13} \to \frac{1}{480},\ \omega_{14} \to 0,$

$\omega_{15} \to -\frac{1}{1008},\ \omega_{16} \to 0,\ \omega_{17} \to \frac{1}{960},\ \omega_{20} \to 0,\ \omega_{21} \to 0,\ \omega_{22} \to \frac{17}{5760},\ \omega_{23} \to 0,\ \omega_{24} \to -\frac{3}{2240},$

$\omega_{25} \to 0,\ \omega_{26} \to \frac{37}{26880},\ \omega_{30} \to 0,\ \omega_{31} \to \frac{1}{480},\ \omega_{32} \to 0,\ \omega_{33} \to -\frac{103}{71680},\ \omega_{34} \to 0,\ \omega_{35} \to \frac{991}{645120},$

$\omega_{40} \to 0,\ \omega_{41} \to 0,\ \omega_{42} \to -\frac{3}{2240},\ \omega_{43} \to 0,\ \omega_{44} \to \frac{20509}{12902400},\ \omega_{50} \to 0,\ \omega_{51} \to -\frac{1}{1008},$

$\omega_{52} \to 0,\ \omega_{53} \to \frac{991}{645120},\ \omega_{60} \to 0,\ \omega_{61} \to 0,\ \omega_{62} \to \frac{37}{26880},\ \omega_{70} \to 0,\ \omega_{71} \to \frac{1}{960},\ \omega_{80} \to 0\}$

{V1, C1} = {V0, C0} /. sol

$$\left\{\begin{pmatrix} 1 - \frac{1}{48}\,(c_1\,c_2)\,\hbar^2 + \left(\frac{c_1^3\,c_2}{2880} + \frac{17\,c_1^2\,c_2^2}{23040} + \frac{c_1\,c_2^3}{2880}\right)\hbar^4 + \left(-\frac{c_1^5\,c_2}{120960} - \frac{c_1^4\,c_2^2}{35840} - \frac{103\,c_1^3\,c_2^3}{2580480} - \frac{c_1^2\,c_2^4}{35840} - \frac{c_1\,c_2^5}{120960}\right)\hbar^6 + \left(\frac{c_1^7\,c_2}{4838400}\right. \\ t[1] \\ t[2] \end{pmatrix}\right.$$

False && Put[{V1, C1, sol}, "SolutionToDegree8-120524.m"]

False