

Pensieve header: The "top" strand deletion for \$V\$.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.0"];
<< betaCalculus.m
Clear[ħ]; Unprotect[C];
$PerturbativeDegree = 6;
βSimplify[expr_] := Replace[
  Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
  sd_SeriesData => MapAt[Expand, sd, 3]
];
βCollect[B[ω_, μ_]] := B[
  βSimplify[ω],
  βSimplify[μ]
];
{V, C, sol} = Get["SolutionToDegree6-120501.m"];
V // ds[2] // dm[1, 2, 1]

```

$$\left(\begin{array}{c} 1 - \frac{c_1 \hbar}{2} + \frac{7}{48} c_1^2 \hbar^2 - \frac{1}{32} c_1^3 \hbar^3 + \frac{121 c_1^4 \hbar^4}{23\,040} - \frac{11 c_1^5 \hbar^5}{15\,360} + \frac{127 c_1^6 \hbar^6}{1\,548\,288} + O[\hbar]^7 \\ t[1] \end{array} \right) \begin{array}{c} h[1] \\ -\frac{1}{2} + \frac{c_1 \hbar}{8} - \frac{1}{48} c_1^2 \hbar^2 + \frac{1}{384} c_1^3 \hbar^3 - \frac{c_1^4 \hbar^4}{3840} + \frac{c_1^5 \hbar^5}{46\,080} - \frac{c_1^6 \hbar^6}{645\,120} + O[\hbar]^7 \end{array}$$

R[1, 1, -1/2]

$$\left(\begin{array}{c} 1 \\ t[1] \end{array} \right) \begin{array}{c} h[1] \\ -\frac{1}{2} + \frac{c_1 \hbar}{8} - \frac{1}{48} c_1^2 \hbar^2 + \frac{1}{384} c_1^3 \hbar^3 - \frac{c_1^4 \hbar^4}{3840} + \frac{c_1^5 \hbar^5}{46\,080} - \frac{c_1^6 \hbar^6}{645\,120} + O[\hbar]^7 \end{array}$$

C

$$\left(\begin{array}{c} 1 + \frac{1}{32} c_1^2 \hbar^2 - \frac{c_1^4 \hbar^4}{2048} - \frac{13 c_1^6 \hbar^6}{44\,236\,800} + O[\hbar]^7 \\ t[1] \end{array} \right)$$

R[1, 1, -1/2] // ds[1]

$$\left(\begin{array}{c} 1 - \frac{c_1 \hbar}{2} + \frac{1}{8} c_1^2 \hbar^2 - \frac{1}{48} c_1^3 \hbar^3 + \frac{1}{384} c_1^4 \hbar^4 - \frac{c_1^5 \hbar^5}{3840} + \frac{c_1^6 \hbar^6}{46\,080} + O[\hbar]^7 \\ t[1] \end{array} \right) \begin{array}{c} h[1] \\ -\frac{1}{2} + \frac{c_1 \hbar}{8} - \frac{1}{48} c_1^2 \hbar^2 + \frac{1}{384} c_1^3 \hbar^3 - \frac{c_1^4 \hbar^4}{3840} + \frac{c_1^5 \hbar^5}{46\,080} \end{array}$$

{V0, C0} = {

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βCollect[
  B[ω[ħ c1, ħ c2], α[ħ c1, ħ c2] t[1] h[1] +
    β[ħ c1, ħ c2] t[1] h[2] + γ[ħ c1, ħ c2] t[2] h[1] + δ[ħ c1, ħ c2] t[2] h[2]]
] /. {
  (ε : (α | β | γ | δ | ω | κ)) [___] => ε0, (ε : (α | β | γ | δ | ω | κ))^(k___) [___] => εFromDigits[{k]}
},
βCollect[B[κ[ħ c1], 0]] /. {
  (ε : (α | β | γ | δ | ω | κ)) [___] => ε0,
  (ε : (α | β | γ | δ | ω | κ))^(k___) [___] => εFromDigits[{k]}
}
} /. sol

```

$$\left\{ \begin{array}{l} 1 + \frac{1}{16} c_1 c_2 (1 + 16 \delta_{10}) \hbar^2 + \left(\frac{1}{256} c_1^2 c_2^2 (-1 + 40 \gamma_1 + 192 \gamma_1^2 - 8 \delta_{10} - 192 \gamma_1 \delta_{10} + 128 \delta_{10}^2 + 96 \delta_{12} + 32 \delta_3) \right. \end{array} \right.$$

Er = v0 // ds[2] // dm[1, 2, 1]

$$\left(\begin{array}{l} 1 - \frac{c_1 \hbar}{2} + \frac{7}{48} c_1^2 \hbar^2 - \frac{1}{32} c_1^3 \hbar^3 + \frac{121 c_1^4 \hbar^4}{23040} - \frac{11 c_1^5 \hbar^5}{15360} + \frac{127 c_1^6 \hbar^6}{1548288} + O[\hbar]^7 \\ t[1] \end{array} \right. \quad \begin{array}{l} h[1] \\ -\frac{1}{2} + \frac{c_1 \hbar}{8} - \frac{1}{48} c_1^2 \hbar^2 + \frac{1}{384} c_1^3 \hbar^3 - \frac{c_1^4 \hbar^4}{3840} + \frac{1}{4} \end{array}$$

Er ** (Er // ds[1])

$$\left(\begin{array}{l} 1 - \frac{c_1 \hbar}{2} + \frac{1}{6} c_1^2 \hbar^2 - \frac{1}{24} c_1^3 \hbar^3 + \frac{1}{120} c_1^4 \hbar^4 - \frac{1}{720} c_1^5 \hbar^5 + \frac{c_1^6 \hbar^6}{5040} + O[\hbar]^7 \\ t[1] \end{array} \right. \quad \begin{array}{l} h[1] \\ -1 + \frac{c_1 \hbar}{2} - \frac{1}{6} c_1^2 \hbar^2 + \frac{1}{24} c_1^3 \hbar^3 - \frac{1}{120} c_1^4 \hbar^4 + \frac{1}{720} c_1^5 \hbar^5 - \frac{c_1^6 \hbar^6}{5040} + O[\hbar]^7 \end{array}$$

R[1, 1, -1] // ds[1]

$$\left(\begin{array}{l} 1 - c_1 \hbar + \frac{1}{2} c_1^2 \hbar^2 - \frac{1}{6} c_1^3 \hbar^3 + \frac{1}{24} c_1^4 \hbar^4 - \frac{1}{120} c_1^5 \hbar^5 + \frac{1}{720} c_1^6 \hbar^6 + O[\hbar]^7 \\ t[1] \end{array} \right. \quad \begin{array}{l} h[1] \\ -1 + \frac{c_1 \hbar}{2} - \frac{1}{6} c_1^2 \hbar^2 + \frac{1}{24} c_1^3 \hbar^3 - \frac{1}{120} c_1^4 \hbar^4 + \frac{1}{720} c_1^5 \hbar^5 - \frac{c_1^6 \hbar^6}{5040} + O[\hbar]^7 \end{array}$$

((Er // ds[1]) ** Er) // dcap[1]

$$\left(\begin{array}{l} 1 + \frac{c_1 \hbar}{2} + \frac{1}{6} c_1^2 \hbar^2 + \frac{1}{24} c_1^3 \hbar^3 + \frac{1}{120} c_1^4 \hbar^4 + \frac{1}{720} c_1^5 \hbar^5 + \frac{c_1^6 \hbar^6}{5040} + O[\hbar]^7 \\ t[1] \end{array} \right)$$

{ser = Series[(E^x - 1) / x, {x, 0, 6}], Exp[ser], Log[ser]} // ColumnForm

$$\begin{aligned} & 1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{24} + \frac{x^4}{120} + \frac{x^5}{720} + \frac{x^6}{5040} + O[x]^7 \\ & e + \frac{e x}{2} + \frac{7 e x^2}{24} + \frac{7 e x^3}{48} + \frac{383 e x^4}{5760} + \frac{109 e x^5}{3840} + \frac{33371 e x^6}{2903040} + O[x]^7 \\ & \frac{x}{2} + \frac{x^2}{24} - \frac{x^4}{2880} + \frac{x^6}{181440} + O[x]^7 \end{aligned}$$

{
Er[[1]] /. c1 -> 1,
Er[[1]] /. c1 -> -1,
(Er[[1]] /. c1 -> 1) (Er[[1]] /. c1 -> -1),
Series[$\frac{\text{Sinh}[\hbar/2]}{\hbar/2}$, {\hbar, 0, 6}],
(Er[[1]] /. c1 -> -1) / (Er[[1]] /. c1 -> 1),
Series[E^{-\hbar/2} $\sqrt{\frac{\text{Sinh}[\hbar/2]}{\hbar/2}}$, {\hbar, 0, 6}]
} // ColumnForm

$$1 - \frac{\hbar}{2} + \frac{7 \hbar^2}{48} - \frac{\hbar^3}{32} + \frac{121 \hbar^4}{23040} - \frac{11 \hbar^5}{15360} + \frac{127 \hbar^6}{1548288} + O[\hbar]^7$$

$$1 + \frac{\hbar}{2} + \frac{7 \hbar^2}{48} + \frac{\hbar^3}{32} + \frac{121 \hbar^4}{23040} + \frac{11 \hbar^5}{15360} + \frac{127 \hbar^6}{1548288} + O[\hbar]^7$$

$$1 + \frac{\hbar^2}{24} + \frac{\hbar^4}{1920} + \frac{\hbar^6}{322560} + O[\hbar]^7$$

$$1 + \frac{\hbar^2}{24} + \frac{\hbar^4}{1920} + \frac{\hbar^6}{322560} + O[\hbar]^7$$

$$1 + \hbar + \frac{\hbar^2}{2} + \frac{\hbar^3}{6} + \frac{\hbar^4}{24} + \frac{\hbar^5}{120} + \frac{\hbar^6}{720} + O[\hbar]^7$$

$$1 - \frac{\hbar}{2} + \frac{7 \hbar^2}{48} - \frac{\hbar^3}{32} + \frac{121 \hbar^4}{23040} - \frac{11 \hbar^5}{15360} + \frac{127 \hbar^6}{1548288} + O[\hbar]^7$$

$$\text{Solve}\left[\mathbf{f} * \mathbf{f} * \mathbf{E}^{\hbar} = \frac{\text{Sinh}[\hbar/2]}{\hbar/2}, \{\mathbf{f}\}\right]$$

$$\left\{\left\{\mathbf{f} \rightarrow -\frac{\sqrt{2} e^{-\hbar/2} \sqrt{\text{Sinh}[\frac{\hbar}{2}]}}{\sqrt{\hbar}}\right\}, \left\{\mathbf{f} \rightarrow \frac{\sqrt{2} e^{-\hbar/2} \sqrt{\text{Sinh}[\frac{\hbar}{2}]}}{\sqrt{\hbar}}\right\}\right\}$$

C0

$$\left(1 + c_1 \kappa_1 \hbar + \frac{1}{32} c_1^2 (1 + 16 \delta_{10} + 16 \kappa_1^2) \hbar^2 + \frac{1}{96} c_1^3 \kappa_1 (3 + 48 \delta_{10} + 16 \kappa_1^2) \hbar^3 + \frac{c_1^4 (-3 + 160 \gamma_1 + 768 \gamma_1^2 - 768 \gamma_1 \delta_{10} + 768 \delta_{10}^2)}{6144}\right)$$

C0 /. {c1 -> 1, kappa1 -> 1/4, delta10 -> -1/48}

$$\left(1 + \frac{\hbar}{4} + \frac{5 \hbar^2}{96} + \frac{\hbar^3}{128} + \frac{\left(\frac{7}{3} + 176 \gamma_1 + 768 \gamma_1^2 + 384 \delta_{12} + 128 \delta_{30}\right) \hbar^4}{6144} + \frac{\left(-\frac{17}{3} + 880 \gamma_1 + 3840 \gamma_1^2 + 1920 \delta_{12} + 640 \delta_{30}\right) \hbar^5}{122880} + \frac{\left(-\frac{1489}{3} + 41936 \gamma_1 - 11520 \gamma_1^2\right) \hbar^6}{122880}\right)$$

$$\text{Series}\left[\sqrt{\frac{\mathbf{E}^{\hbar} - 1}{\hbar}}, \{\hbar, 0, 6\}\right]$$

$$1 + \frac{\hbar}{4} + \frac{5 \hbar^2}{96} + \frac{\hbar^3}{128} + \frac{79 \hbar^4}{92160} + \frac{3 \hbar^5}{40960} + \frac{71 \hbar^6}{12386304} + O[\hbar]^7$$

$$\text{SolveAlways}\left[\text{Normal}\left[\text{Series}\left[\sqrt{\frac{\mathbf{E}^{\hbar} - 1}{\hbar}}, \{\hbar, 0, 6\}\right] - (\text{C0}[[1]] /. c_1 \rightarrow 1)\right] == 0, \{\hbar\}\right]$$

$$\left\{\left\{\delta_{23} \rightarrow \frac{1}{322560} (1391 - 49560 \gamma_1 - 1370880 \gamma_1^2 + 7741440 \gamma_1^3 - 443520 \gamma_{12} - 3870720 \gamma_1 \gamma_{12} - 147840 \gamma_{30} - 1290240 \gamma_1 \gamma_{30} - 161280 \delta_{41}), \delta_{12} \rightarrow \frac{11 - 660 \gamma_1 - 2880 \gamma_1^2 - 480 \delta_{30}}{1440}, \delta_{10} \rightarrow -\frac{1}{48}, \kappa_1 \rightarrow \frac{1}{4}\right\}\right\}$$

SolveAlways[Normal[Series[Cosh[ħ], {ħ, 0, 6}] - (C0[[1]] /. c1 -> 1)] == 0, {ħ}]

$$\left\{\left\{\delta_{23} \rightarrow \frac{1}{15360} (-2489 - 84472 \gamma_1 - 241920 \gamma_1^2 + 368640 \gamma_1^3 + 67200 \gamma_{12} - 184320 \gamma_1 \gamma_{12} + 22400 \gamma_{30} - 61440 \gamma_1 \gamma_{30} - 7680 \delta_{41}), \delta_{12} \rightarrow \frac{1}{24} (-26 + 35 \gamma_1 - 48 \gamma_1^2 - 8 \delta_{30}), \delta_{10} \rightarrow \frac{15}{16}, \kappa_1 \rightarrow 0\right\}\right\}$$

SolveAlways[Normal[Series[1, {h, 0, 6}] - (C0[[1]] /. c1 -> 1)] == 0, {h}]

$$\left\{ \left\{ \delta_{23} \rightarrow \frac{1}{15360} (71 - 3832 \gamma_1 - 57600 \gamma_1^2 + 368640 \gamma_1^3 - 24960 \gamma_{12} - 184320 \gamma_1 \gamma_{12} - 8320 \gamma_{30} - 61440 \gamma_1 \gamma_{30} - 7680 \delta_{41}), \right. \right.$$

$$\delta_{12} \rightarrow \frac{1}{24} (-13 \gamma_1 - 48 \gamma_1^2 - 8 \delta_{30}), \delta_{10} \rightarrow -\frac{1}{16}, \kappa_1 \rightarrow 0 \left. \right\},$$

$$\left\{ \delta_{23} \rightarrow \frac{1}{15360} (71 - 3832 \gamma_1 - 57600 \gamma_1^2 + 368640 \gamma_1^3 - 24960 \gamma_{12} - 184320 \gamma_1 \gamma_{12} - 8320 \gamma_{30} - 61440 \gamma_1 \gamma_{30} - 7680 \delta_{41}), \right.$$

$$\delta_{12} \rightarrow \frac{1}{24} (-13 \gamma_1 - 48 \gamma_1^2 - 8 \delta_{30}), \delta_{10} \rightarrow -\frac{1}{16}, \kappa_1 \rightarrow 0 \left. \right\},$$

$$\left\{ \delta_{23} \rightarrow \frac{1}{15360} (71 - 3832 \gamma_1 - 57600 \gamma_1^2 + 368640 \gamma_1^3 - 24960 \gamma_{12} - 184320 \gamma_1 \gamma_{12} - 8320 \gamma_{30} - 61440 \gamma_1 \gamma_{30} - 7680 \delta_{41}), \right.$$

$$\delta_{12} \rightarrow \frac{1}{24} (-13 \gamma_1 - 48 \gamma_1^2 - 8 \delta_{30}), \delta_{10} \rightarrow -\frac{1}{16}, \kappa_1 \rightarrow 0 \left. \right\},$$

$$\left\{ \delta_{23} \rightarrow \frac{1}{15360} (71 - 3832 \gamma_1 - 57600 \gamma_1^2 + 368640 \gamma_1^3 - 24960 \gamma_{12} - 184320 \gamma_1 \gamma_{12} - 8320 \gamma_{30} - 61440 \gamma_1 \gamma_{30} - 7680 \delta_{41}), \right.$$

$$\delta_{12} \rightarrow \frac{1}{24} (-13 \gamma_1 - 48 \gamma_1^2 - 8 \delta_{30}), \delta_{10} \rightarrow -\frac{1}{16}, \kappa_1 \rightarrow 0 \left. \right\}$$

rules = {kappa1 -> 0, delta10 -> -1/16};

((V0 /. rules) // Inverse) ((C0 /. rules) // dP[3]) // dm[2, 3, 2] // dcap[2] // teta[1]

$$\left(1 + \left(\frac{13}{384} c_2^4 \gamma_1 + \frac{1}{8} c_2^4 \gamma_1^2 + \frac{1}{16} c_2^4 \delta_{12} + \frac{1}{48} c_2^4 \delta_{30} \right) h^4 + \left(-\frac{71 c_2^6}{5529600} - \frac{59 c_2^6 \gamma_1}{172800} - \frac{53 c_2^6 \gamma_1^2}{5760} - \frac{1}{8} c_2^6 \gamma_1^3 + \frac{13 c_2^6 \gamma_{12}}{2880} + \frac{1}{30} c_2^6 \right. \right.$$

sol = **SolveAlways**[

((V0 // Inverse) (C0 // dP[3])) // dm[2, 3, 2] // dcap[2] // teta[1] == B[1, 0],

{h, c2}

]

$$\left\{ \left\{ \gamma_{50} \rightarrow -\frac{31}{8064}, \delta_{23} \rightarrow \frac{31 - 74880 \gamma_{12} - 23040 \delta_{41}}{46080}, \right. \right.$$

$$\gamma_{30} \rightarrow \frac{7}{960}, \delta_{12} \rightarrow -\frac{\delta_{30}}{3}, \delta_{10} \rightarrow -\frac{1}{16}, \gamma_1 \rightarrow 0, \kappa_1 \rightarrow 0 \left. \right\}, \left\{ \gamma_{50} \rightarrow -\frac{31}{8064}, \right.$$

$$\delta_{23} \rightarrow \frac{31 - 74880 \gamma_{12} - 23040 \delta_{41}}{46080}, \gamma_{30} \rightarrow \frac{7}{960}, \delta_{12} \rightarrow -\frac{\delta_{30}}{3}, \delta_{10} \rightarrow -\frac{1}{16}, \gamma_1 \rightarrow 0, \kappa_1 \rightarrow 0 \left. \right\},$$

$$\left\{ \gamma_{50} \rightarrow -\frac{31}{8064}, \delta_{23} \rightarrow \frac{31 - 74880 \gamma_{12} - 23040 \delta_{41}}{46080}, \gamma_{30} \rightarrow \frac{7}{960}, \delta_{12} \rightarrow -\frac{\delta_{30}}{3}, \right.$$

$$\delta_{10} \rightarrow -\frac{1}{16}, \gamma_1 \rightarrow 0, \kappa_1 \rightarrow 0 \left. \right\}, \left\{ \gamma_{50} \rightarrow -\frac{31}{8064}, \delta_{23} \rightarrow \frac{31 - 74880 \gamma_{12} - 23040 \delta_{41}}{46080}, \right.$$

$$\gamma_{30} \rightarrow \frac{7}{960}, \delta_{12} \rightarrow -\frac{\delta_{30}}{3}, \delta_{10} \rightarrow -\frac{1}{16}, \gamma_1 \rightarrow 0, \kappa_1 \rightarrow 0 \left. \right\}$$

{sol} = **Union**[sol]

$$\left\{ \left\{ \gamma_{50} \rightarrow -\frac{31}{8064}, \delta_{23} \rightarrow \frac{31 - 74880 \gamma_{12} - 23040 \delta_{41}}{46080}, \right. \right.$$

$$\gamma_{30} \rightarrow \frac{7}{960}, \delta_{12} \rightarrow -\frac{\delta_{30}}{3}, \delta_{10} \rightarrow -\frac{1}{16}, \gamma_1 \rightarrow 0, \kappa_1 \rightarrow 0 \left. \right\}$$

{v1, c1} = {v0, c0} /. sol

$$\left\{ 1 + \left(\frac{c_1^5 c_2 \left(-\frac{31}{3} + 24960 \gamma_{12} + \frac{1}{3} (31 - 74880 \gamma_{12} - 23040 \delta_{41}) + 7680 \delta_{41} \right)}{921600} + \frac{c_1^4 c_2^2 \left(-\frac{31}{3} + 24960 \gamma_{12} + \frac{1}{3} (31 - 74880 \gamma_{12} - 23040 \delta_{41}) + 7680 \delta_{41} \right)}{368640} \right) + \right.$$

$\Phi_1 = \Phi[v1]$

$$\left\{ \begin{array}{l} 1 \\ t[1] \\ t[2] - \frac{c_3 \hbar}{16} + \left(\frac{1}{720} c_1^2 c_3 + \frac{c_1 c_2 c_3}{1152} + \frac{1}{384} c_2^2 c_3 - \frac{5 c_1 c_3^2}{4608} + \frac{13 c_2 c_3^2}{4608} + \frac{1}{2} c_1^2 c_3 \gamma_{12} + c_1 c_2 c_3 \gamma_{12} + \frac{1}{2} c_1 c_3^2 \gamma_{12} + \frac{1}{6} c_3 \right) \\ t[3] \end{array} \right.$$

$\Phi_1 // d\Delta[1, 0, 1] // ds[2] // ds[3] // dm[0, 3, 0] // dm[1, 2, 1]$

$$\left\{ 1 - \frac{1}{48} (c_0 c_1) \hbar^2 + \left(-\frac{c_0^3 c_1}{1440} - \frac{13 c_0^2 c_1^2}{7680} - \frac{1}{960} c_0 c_1^3 - \frac{1}{2} c_0^2 c_1^2 \gamma_{12} - \frac{1}{2} c_0 c_1^3 \gamma_{12} - \frac{1}{6} c_0^3 c_1 \delta_{30} - \frac{1}{2} c_0^2 c_1^2 \delta_{30} - \frac{1}{3} c_0 c_1^3 \delta_{30} \right) \right.$$