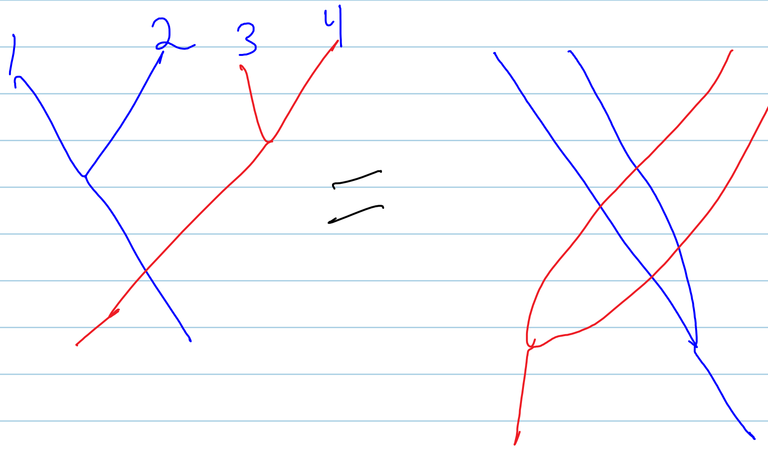
Scratch for "Foundations"

April-28-12
6:19 PM

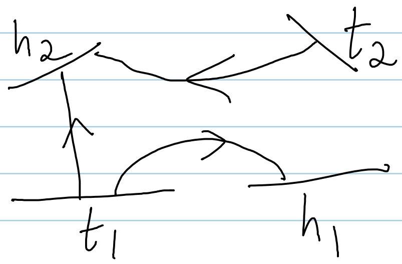



```
In[23]:= {b = B[ω, α t[1] h[1] + β t[1] h[2] + δ t[2] h[2]],
  b // thswap[1, 1],
  b // Wheel // thswap[1, 1] // DeWheel
}
```

Out[23]= $\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & 0 & \delta \end{pmatrix}, \begin{pmatrix} \omega + \alpha\,\omega\,c_1 & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & 0 & \delta \end{pmatrix}, \begin{pmatrix} e^{\alpha c_1}\,\omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & 0 & \delta \end{pmatrix}\right\}$

Zeros stay!

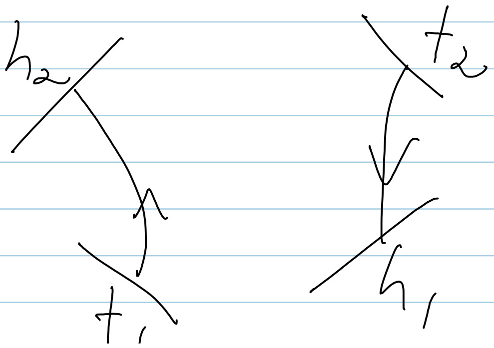
Above:

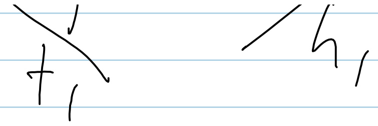



```
In[26]:= {b = B[ω, β t[1] h[2] + γ t[2] h[1]],
  b // thswap[1, 1],
  b // Wheel // thswap[1, 1] // DeWheel
}
```

Out[26]= $\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & 0 & \beta \\ t[2] & \gamma & 0 \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & 0 & \beta + \beta\,\gamma\,c_2 \\ t[2] & \gamma & -\beta\,\gamma\,c_1 \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & 0 & e^{\gamma c_2}\,\beta \\ t[2] & \gamma & -\frac{\left(-1+e^{\gamma c_2}\right)\beta\,c_1}{c_2} \end{pmatrix}\right\}$

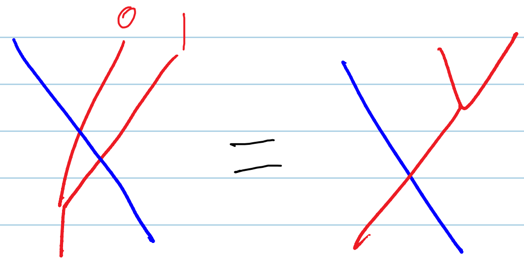
Above





Conclusion: better factorize heads!

$$\begin{pmatrix}\alpha & \beta\\ \gamma & 0\end{pmatrix} \sim \begin{array}{c}\begin{array}{ccc}0 & 1 & 2\end{array}\\ \begin{pmatrix}\alpha & 0 & \beta\\ 0 & \gamma & 0\end{pmatrix}\end{array} // hm^{0,1}_{1}$$



This works:

In[35]:=
```
{b = B[ω, α t[1] h[0] + γ / (1 + α c1) t[2] h[1] + β t[1] h[2]],
  b // hm[0, 1, 1],
  b // hm[0, 1, 1] // thswap[1, 1],
  b // thswap[1, 0],
  b // thswap[1, 0] // thswap[1, 1],
  b // thswap[1, 0] // thswap[1, 1] // hm[0, 1, 1]
}
```

Out[35]=

$$\left\{\begin{pmatrix}\omega & h[0] & h[1] & h[2]\\ t[1] & \alpha & 0 & \beta\\ t[2] & 0 & \frac{\gamma}{1+\alpha c_1} & 0\end{pmatrix},\ \begin{pmatrix}\omega & h[1] & h[2]\\ t[1] & \alpha & \beta\\ t[2] & \gamma & 0\end{pmatrix},\right.$$

$$\begin{pmatrix}\omega+\alpha\omega c_1 & h[1] & h[2]\\ t[1] & \alpha+\frac{\alpha\gamma c_2}{1+\alpha c_1} & \beta+\frac{\beta\gamma c_2}{1+\alpha c_1}\\ t[2] & \frac{\gamma}{1+\alpha c_1} & -\frac{\beta\gamma c_1}{1+\alpha c_1}\end{pmatrix},\ \begin{pmatrix}\omega+\alpha\omega c_1 & h[0] & h[1] & h[2]\\ t[1] & \alpha & 0 & \beta\\ t[2] & 0 & \frac{\gamma}{1+\alpha c_1} & 0\end{pmatrix},$$

$$\left.\begin{pmatrix}\omega+\alpha\omega c_1 & h[0] & h[1] & h[2]\\ t[1] & \alpha+\frac{\alpha\gamma c_2}{1+\alpha c_1} & 0 & \beta+\frac{\beta\gamma c_2}{1+\alpha c_1}\\ t[2] & \gamma\left(-1+\frac{1}{1+\alpha c_1}\right) & \frac{\gamma}{1+\alpha c_1} & -\frac{\beta\gamma c_1}{1+\alpha c_1}\end{pmatrix},\ \begin{pmatrix}\omega+\alpha\omega c_1 & h[1] & h[2]\\ t[1] & \alpha+\frac{\alpha\gamma c_2}{1+\alpha c_1} & \beta+\frac{\beta\gamma c_2}{1+\alpha c_1}\\ t[2] & \frac{\gamma}{1+\alpha c_1} & -\frac{\beta\gamma c_1}{1+\alpha c_1}\end{pmatrix}\right\}$$