Burau Calculus

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11:40 AM

From 2012-03: BurauCalculus.nb:

```
n = 5;
{
   β1 = Bu[ω, Table[h[j] → λj, {j, n}],
     Sum[α10 i+j t[i] h[j] + (1 / n - α10 i+j) t[j] h[j], {i, n}, {j, n}]
    ],
   B[β1],
   (Expand /@ Bu[n, B[β1] // dm[1, 2, 1]]) /. t[i_] h[i_] → 0
  } // βForm // ColumnForm
```

ω	h[1] → λ1	h[2] → λ2	h[3] → λ3	h[4] → λ4	h[5] → λ5
t[1]	1 - α21 - α31 - α41 - α51	α12	α13	α14	α15
t[2]	α21	1 - α12 - α32 - α42 - α52	α23	α24	α25
t[3]	α31	α32	1 - α13 - α23 - α43 - α53	α34	α35
t[4]	α41	α42	α43	1 - α14 - α24 - α34 - α54	α45
t[5]	α51	α52	α53	α54	1 - α15 - α25 - α35 - α45

ω	h[1]	h[2]	h[3]	h[4]	h[5]
t[1]	-1 + α21 + α31 + α41 + α51 + λ1	-α12	-α13	-α14	-α15
t[2]	-α21	-1 + α12 + α32 + α42 + α52 + λ2	-α23	-α24	-α25
t[3]	-α31	-α32	-1 + α13 + α23 + α43 + α53 + λ3	-α34	-α35
t[4]	-α41	-α42	-α43	-1 + α14 + α24 + α34 + α54 + λ4	-α45
t[5]	-α51	-α52	-α53	-α54	-1 + α15 + α25 + α35 + α45 + λ5

ω - ω α12	h[1] → λ1 λ2	h[2] → 1	h[3] → λ3	h[4] → λ4	h[5] → λ5
t[1]	0	0	$\frac{(-1+\alpha_{12})\,\alpha_{23}+\alpha_{13}\,(-1+\alpha_{12}+\alpha_{32}+\alpha_{42}+\alpha_{52})}{-1+\alpha_{12}}$	$\frac{(-1+\alpha_{12})\,\alpha_{24}+\alpha_{14}\,(-1+\alpha_{12}+\alpha_{32}+\alpha_{42}+\alpha_{52})}{-1+\alpha_{12}}$	$\frac{(-1+\alpha_{12})\,\alpha_{25}+\alpha_{15}\,(-1+\alpha_{12}+\alpha_{32}+\alpha_{42}+\alpha_{52})}{-1+\alpha_{12}}$
t[3]	$\frac{\alpha_{31}\,(-1+\alpha_{12}+\alpha_{32})+\alpha_{32}\,(-1+\alpha_{21}+\alpha_{41}+\alpha_{51})}{-1+\alpha_{12}}$	0	0	$-\frac{\alpha_{14}\,\alpha_{32}}{-1+\alpha_{12}}+\alpha_{34}$	$-\frac{\alpha_{15}\,\alpha_{32}}{-1+\alpha_{12}}+\alpha_{35}$
t[4]	$\frac{\alpha_{41}\,(-1+\alpha_{12}+\alpha_{42})+\alpha_{42}\,(-1+\alpha_{21}+\alpha_{31}+\alpha_{51})}{-1+\alpha_{12}}$	0	$-\frac{\alpha_{13}\,\alpha_{42}}{-1+\alpha_{12}}+\alpha_{43}$	0	$-\frac{\alpha_{15}\,\alpha_{42}}{-1+\alpha_{12}}+\alpha_{45}$
t[5]	$\frac{(-1+\alpha_{21}+\alpha_{31}+\alpha_{41})\,\alpha_{52}+\alpha_{51}\,(-1+\alpha_{12}+\alpha_{52})}{-1+\alpha_{12}}$	0	$-\frac{\alpha_{13}\,\alpha_{52}}{-1+\alpha_{12}}+\alpha_{53}$	$-\frac{\alpha_{14}\,\alpha_{52}}{-1+\alpha_{12}}+\alpha_{54}$	0

$$\begin{bmatrix} 0 & \alpha & \beta \\ \gamma & 0 & \delta \\ \epsilon & \eta & \theta \end{bmatrix} \xrightarrow{m_{1,2}} \begin{bmatrix} 0 & \beta+\delta+\beta\frac{\langle\eta\rangle}{\alpha-1} \\ \epsilon+\eta\left(\frac{\gamma+\langle\epsilon\rangle}{\alpha-1}\right) & \theta-\frac{\eta\cdot\beta}{\alpha-1} \end{bmatrix}$$

From 2012-04: BurauCalculus.nb:

```
⟨α_⟩ := α /. t[_] → 1;
m[x_, y_, z_][Bu[ω_, μ_]] := Module[
   {α, β, γ, δ, ϵ, η, θ},
   α = Coefficient[μ, t[x] h[y]];
   β = Coefficient[μ, t[x]] /. h[y] → 0;
   γ = Coefficient[μ, t[y] h[x]];
   δ = Coefficient[μ, t[y]] /. h[x] → 0;
   ϵ = Coefficient[μ, h[x]] /. t[y] → 0;
   η = Coefficient[μ, h[y]] /. t[x] → 0;
   θ = μ /. t[x] | t[y] | h[x] | h[y] → 0;
   (Bu[ω (1 - α),
       t[z] (δ + β (1 + ⟨η⟩ / (α - 1))) + h[z] (ϵ + η (γ + ⟨ϵ⟩) / (α - 1))
        + θ - (Expand[η * β] /. t[s_] h[s_] → 0) / (α - 1)
      ] /. Tx | Ty → Tz) // BuCollect
  ];
Bu /: Bu[ω1_, μ1_] * Bu[ω2_, μ2_] := Bu[ω1 * ω2, μ1 + μ2];
Rp[x_, y_] := Bu[1, t[x] h[y] (1 - Tx)];
Rm[x_, y_] := Bu[1, t[x] h[y] (1 - 1 / Tx)];
```

remove diagonal.

with $\alpha' = w\alpha$; $\gamma = \frac{\alpha'}{\omega}$ etc., get

$$w \xrightarrow{m_1^{12}} w(1-\alpha) = w\left(1-\frac{\alpha'}{w}\right) = w-\alpha' = \tilde{w}$$

$$\delta+\beta\left(1+\frac{\langle\eta\rangle}{\alpha-1}\right) = \frac{\delta'}{w}+\frac{\beta'}{w}\left(1+\frac{\langle\eta'/w\rangle}{\alpha'/w-1}\right)$$

$$\longrightarrow (w-\alpha')\left(\frac{\delta'}{w}+\frac{\beta'}{w}\left(1+\frac{\langle\eta'/w\rangle}{\alpha'/w-1}\right)\right)$$

$=$