Pensieve header: Some foundational calculations for β-calculus.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-04"];
<< betaCalculus.m
```

```
βSimplify = FullSimplify;
Unprotect[Log]; Log[E^x_] := x;
K[B[ω_, μ_]] := Module[{heads, ξs, nheads},
   heads = hL[B[ω, μ]];
   ξs = (D[μ, h[#]] /. t[s_] :→ c_s) & /@ heads;
   nheads = MapThread[(h[#1] * Log[1 + #2] / #2) &, {heads, ξs}];
   βCollect[B[ω, μ] /. Thread[(h /@ heads) → nheads]]
  ];
J[B[ω_, μ_]] := Module[{heads, ηs, nheads},
   heads = hL[B[ω, μ]];
   ηs = (D[μ, h[#]] /. t[s_] :→ c_s) & /@ heads;
   nheads = MapThread[(h[#1] * (Exp[#2] - 1) / #2) &, {heads, ηs}];
   βCollect[B[ω, μ] /. Thread[(h /@ heads) → nheads]]
  ];
```

```
{b = B[ω, α t[1] h[1] + β t[1] h[2] + γ t[2] h[1] + δ t[2] h[2]],
 b // J,
 b // J // K,
 b // K,
 b // K // J
}
```

$$\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & \delta \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \frac{\left(-1+e^{\alpha c_1+\gamma c_2}\right)\alpha}{\alpha c_1+\gamma c_2} & \frac{\left(-1+e^{\beta c_1+\delta c_2}\right)\beta}{\beta c_1+\delta c_2} \\ t[2] & \frac{\left(-1+e^{\alpha c_1+\gamma c_2}\right)\gamma}{\alpha c_1+\gamma c_2} & \frac{\left(-1+e^{\beta c_1+\delta c_2}\right)\delta}{\beta c_1+\delta c_2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & \delta \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \frac{\alpha \operatorname{Log}[1+\alpha c_1+\gamma c_2]}{\alpha c_1+\gamma c_2} & \frac{\beta \operatorname{Log}[1+\beta c_1+\delta c_2]}{\beta c_1+\delta c_2} \\ t[2] & \frac{\gamma \operatorname{Log}[1+\alpha c_1+\gamma c_2]}{\alpha c_1+\gamma c_2} & \frac{\delta \operatorname{Log}[1+\beta c_1+\delta c_2]}{\beta c_1+\delta c_2} \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & \delta \end{pmatrix}\right\}$$

```
{b = B[ω, α t[1] h[1] + β t[2] h[2]],
 b // hm[1, 2, 1],
 b // J,
 b // J // hm[1, 2, 1],
 b // J // hm[1, 2, 1] // K
}
```

$$\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & 0 \\ t[2] & 0 & \beta \end{pmatrix}, \begin{pmatrix} \omega & h[1] \\ t[1] & \alpha \\ t[2] & \beta+\alpha\beta c_1 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \frac{-1+e^{\alpha c_1}}{c_1} & 0 \\ t[2] & 0 & \frac{-1+e^{\beta c_2}}{c_2} \end{pmatrix}, \begin{pmatrix} \omega & h[1] \\ t[1] & \frac{-1+e^{\alpha c_1}}{c_1} \\ t[2] & \frac{e^{\alpha c_1}\left(-1+e^{\beta c_2}\right)}{c_2} \end{pmatrix}, \begin{pmatrix} \omega & h[1] \\ t[1] & \frac{\left(-1+e^{\alpha c_1}\right)(\alpha c_1+\beta c_2)}{\left(-1+e^{\alpha c_1+\beta c_2}\right) c_1} \\ t[2] & \frac{e^{\alpha c_1}\left(-1+e^{\beta c_2}\right)(\alpha c_1+\beta c_2)}{\left(-1+e^{\alpha c_1+\beta c_2}\right) c_2} \end{pmatrix}\right\}$$

```
{b = B[ω, α t[1] h[1] + β t[1] h[2] + γ t[2] h[1] + δ t[2] h[2]],
 b // hm[1, 2, 1],
 b // J,
 b // J // hm[1, 2, 1],
 b // J // hm[1, 2, 1] // K
}
```

$$\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & \delta \end{pmatrix}, \begin{pmatrix} \omega & h[1] \\ t[1] & \alpha+\beta+\alpha\beta c_1+\beta\gamma c_2 \\ t[2] & \gamma+\delta+\alpha\delta c_1+\gamma\delta c_2 \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \frac{(-1+e^{\alpha c_1+\gamma c_2})\alpha}{\alpha c_1+\gamma c_2} & \frac{(-1+e^{\beta c_1+\delta c_2})\beta}{\beta c_1+\delta c_2} \\ t[2] & \frac{(-1+e^{\alpha c_1+\gamma c_2})\gamma}{\alpha c_1+\gamma c_2} & \frac{(-1+e^{\beta c_1+\delta c_2})\delta}{\beta c_1+\delta c_2} \end{pmatrix},\right.$$

$$\begin{pmatrix} \omega & h[1] \\ t[1] & \frac{(-1+e^{(\alpha+\beta)c_1+(\gamma+\delta)c_2})\alpha\beta c_1+(-\alpha\delta+e^{\alpha c_1+\gamma c_2}((-1+e^{\beta c_1+\delta c_2})\beta\gamma+\alpha\delta))c_2}{(\alpha c_1+\gamma c_2)(\beta c_1+\delta c_2)} \\ t[2] & \frac{((-1+e^{\alpha c_1+\gamma c_2})\beta\gamma+e^{\alpha c_1+\gamma c_2}(-1+e^{\beta c_1+\delta c_2})\alpha\delta)c_1+(-1+e^{(\alpha+\beta)c_1+(\gamma+\delta)c_2})\gamma\delta c_2}{(\alpha c_1+\gamma c_2)(\beta c_1+\delta c_2)} \end{pmatrix},$$

$$\left.\begin{pmatrix} \omega & h[1] \\ t[1] & \frac{((\alpha+\beta)c_1+(\gamma+\delta)c_2)((-1+e^{(\alpha+\beta)c_1+(\gamma+\delta)c_2})\alpha\beta c_1+(-\alpha\delta+e^{\alpha c_1+\gamma c_2}((-1+e^{\beta c_1+\delta c_2})\beta\gamma+\alpha\delta))c_2)}{(-1+e^{(\alpha+\beta)c_1+(\gamma+\delta)c_2})(\alpha c_1+\gamma c_2)(\beta c_1+\delta c_2)} \\ t[2] & \frac{(((-1+e^{\alpha c_1+\gamma c_2})\beta\gamma+e^{\alpha c_1+\gamma c_2}(-1+e^{\beta c_1+\delta c_2})\alpha\delta)c_1+(-1+e^{(\alpha+\beta)c_1+(\gamma+\delta)c_2})\gamma\delta c_2)((\alpha+\beta)c_1+(\gamma+\delta)c_2)}{(-1+e^{(\alpha+\beta)c_1+(\gamma+\delta)c_2})(\alpha c_1+\gamma c_2)(\beta c_1+\delta c_2)} \end{pmatrix}\right\}$$

```
{b = B[ω, α t[1] h[1] + β t[1] h[2] + γ t[2] h[1] + δ t[2] h[2]],
 b // thswap[1, 1],
 b // J // thswap[1, 1] // K
}
```

$$\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & \delta \end{pmatrix}, \begin{pmatrix} \omega+\alpha\omega c_1 & h[1] & h[2] \\ t[1] & \alpha+\frac{\alpha\gamma c_2}{1+\alpha c_1} & \beta+\frac{\beta\gamma c_2}{1+\alpha c_1} \\ t[2] & \frac{\gamma}{1+\alpha c_1} & \delta-\frac{\beta\gamma c_1}{1+\alpha c_1} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \omega\left(1+\frac{(-1+e^{\alpha c_1+\gamma c_2})\alpha c_1}{\alpha c_1+\gamma c_2}\right) & h[1] & h[2] \\ t[1] & \frac{e^{\alpha c_1+\gamma c_2}\alpha(\alpha c_1+\gamma c_2)}{e^{\alpha c_1+\gamma c_2}\alpha c_1+\gamma c_2} & \frac{e^{\alpha c_1+\gamma c_2}\beta(\alpha c_1+\gamma c_2)}{e^{\alpha c_1+\gamma c_2}\alpha c_1+\gamma c_2} \\ t[2] & \frac{\gamma(\alpha c_1+\gamma c_2)}{e^{\alpha c_1+\gamma c_2}\alpha c_1+\gamma c_2} & \frac{-\beta\gamma+\alpha\delta+\frac{\beta\gamma(\alpha c_1+\gamma c_2)}{e^{\alpha c_1+\gamma c_2}\alpha c_1+\gamma c_2}}{\alpha} \end{pmatrix}\right\}$$

```
{b = B[ω, α t[1] h[1] + β t[1] h[2] + δ t[2] h[2]],
 b // thswap[1, 1],
 b // J // thswap[1, 1] // K
}
```

$$\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & 0 & \delta \end{pmatrix}, \begin{pmatrix} \omega+\alpha\omega c_1 & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & 0 & \delta \end{pmatrix}, \begin{pmatrix} e^{\alpha c_1}\omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & 0 & \delta \end{pmatrix}\right\}$$

```
{b = B[ω, β t[1] h[2] + γ t[2] h[1]],
 b // thswap[1, 1],
 b // J // thswap[1, 1] // K
}
```

$$\left\{\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & 0 & \beta \\ t[2] & \gamma & 0 \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & 0 & \beta+\beta\gamma c_2 \\ t[2] & \gamma & -\beta\gamma c_1 \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & 0 & e^{\gamma c_2}\beta \\ t[2] & \gamma & -\frac{(-1+e^{\gamma c_2})\beta c_1}{c_2} \end{pmatrix}\right\}$$

```
{b = B[ω, α t[1] h[0] + γ / (1 + α c₁) t[2] h[1] + β t[1] h[2]],
 b // hm[0, 1, 1],
 b // hm[0, 1, 1] // thswap[1, 1],
 b // thswap[1, 0],
 b // thswap[1, 0] // thswap[1, 1],
 b // thswap[1, 0] // thswap[1, 1] // hm[0, 1, 1]
}
```

$$\left\{\begin{pmatrix} \omega & h[0] & h[1] & h[2] \\ t[1] & \alpha & 0 & \beta \\ t[2] & 0 & \frac{\gamma}{1+\alpha c_1} & 0 \end{pmatrix}, \begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & 0 \end{pmatrix},\right.$$

$$\begin{pmatrix} \omega + \alpha\,\omega\,c_1 & h[1] & h[2] \\ t[1] & \alpha + \frac{\alpha\gamma c_2}{1+\alpha c_1} & \beta + \frac{\beta\gamma c_2}{1+\alpha c_1} \\ t[2] & \frac{\gamma}{1+\alpha c_1} & -\frac{\beta\gamma c_1}{1+\alpha c_1} \end{pmatrix}, \begin{pmatrix} \omega + \alpha\,\omega\,c_1 & h[0] & h[1] & h[2] \\ t[1] & \alpha & 0 & \beta \\ t[2] & 0 & \frac{\gamma}{1+\alpha c_1} & 0 \end{pmatrix},$$

$$\left.\begin{pmatrix} \omega + \alpha\,\omega\,c_1 & h[0] & h[1] & h[2] \\ t[1] & \alpha + \frac{\alpha\gamma c_2}{1+\alpha c_1} & 0 & \beta + \frac{\beta\gamma c_2}{1+\alpha c_1} \\ t[2] & \gamma\left(-1 + \frac{1}{1+\alpha c_1}\right) & \frac{\gamma}{1+\alpha c_1} & -\frac{\beta\gamma c_1}{1+\alpha c_1} \end{pmatrix}, \begin{pmatrix} \omega + \alpha\,\omega\,c_1 & h[1] & h[2] \\ t[1] & \alpha + \frac{\alpha\gamma c_2}{1+\alpha c_1} & \beta + \frac{\beta\gamma c_2}{1+\alpha c_1} \\ t[2] & \frac{\gamma}{1+\alpha c_1} & -\frac{\beta\gamma c_1}{1+\alpha c_1} \end{pmatrix}\right\}$$