

Pensieve Header: β -calculus with controlled column sums.

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 $\beta\text{Simplify} = \text{Factor};$ 
 $\text{SetAttributes}[\beta\text{Collect}, \text{Listable}];$ 
 $\beta\text{Collect}[B[w_, \sigma_, \mu_]] := B[$ 
 $\quad \beta\text{Simplify}[w], \sigma,$ 
 $\quad \text{Collect}[\mu, _h, \text{Collect}[\#, _t, \beta\text{Simplify}] \&]$ 
 $\quad];$ 
 $(* "L" for "Labels" *)$ 
 $hL[B[w_, \sigma_, \mu_]] := \text{Union}[\text{Cases}[\{\sigma\}, h[s_] \rightarrow s, \text{Infinity}]];$ 
 $tL[\beta_] := \text{Union}[\text{Cases}[\beta, t[s_] | T_s_ \rightarrow s, \text{Infinity}]];$ 
 $dL[\beta_] := \text{Union}[hL[\beta], tL[\beta]];$ 
 $\beta\text{Form}[B[w_, \sigma_, \mu_]] := \text{Module}[$ 
 $\quad \{tails, heads, mat\},$ 
 $\quad tails = tL[B[w, \sigma, \mu]]; heads = hL[B[w, \sigma, \mu]];$ 
 $\quad mat = \text{Outer}[\beta\text{Simplify}[\text{Coefficient}[\mu, h[\#1] t[\#2]]] \&, heads, tails];$ 
 $\quad \text{PrependTo}[mat, t /@ tails];$ 
 $\quad mat = \text{Join}[$ 
 $\quad \quad \{\text{Prepend}[h /@ heads, w]\},$ 
 $\quad \quad \text{Transpose}[mat],$ 
 $\quad \quad \{\text{Prepend}[D[\sigma, h[\#]] \& /@ heads, "1+\Sigma/\omega"]\}$ 
 $\quad \};$ 
 $\quad \text{MatrixForm}[mat]$ 
 $\quad];$ 
 $\beta\text{Form}[else_] := else /. \beta_B \rightarrow \beta\text{Form}[\beta];$ 
 $tm[x_, y_, z_][\beta_] := \beta /. \{t[x] \rightarrow t[z], t[y] \rightarrow t[z], T_x \rightarrow T_z, T_y \rightarrow T_z\};$ 
 $hm[x_, y_, z_][B[w_, \sigma_, \mu_]] := B[w,$ 
 $\quad h[z] D[\sigma, h[x]] D[\sigma, h[y]] + (\sigma /. h[x] | h[y] \rightarrow 0),$ 
 $\quad h[z] (D[\mu, h[x]] + D[\sigma, h[x]] D[\mu, h[y]]) + (\mu /. h[x] | h[y] \rightarrow 0)$ 
 $\quad] // \beta\text{Collect};$ 
 $swapth[y_, x_][B[w_, \sigma_, \mu_]] := \text{Module}[$ 
 $\quad \{\alpha, \beta, \gamma, \delta\},$ 
 $\quad \alpha = \text{Coefficient}[\mu, h[x] t[y]]; \beta = D[\mu, t[y]] /. h[x] \rightarrow 0;$ 
 $\quad \gamma = D[\mu, h[x]] /. t[y] \rightarrow 0; \delta = \mu /. h[x] | t[y] \rightarrow 0;$ 
 $\quad B[w + \alpha, \sigma, \text{Plus}[$ 
 $\quad \quad D[\sigma, h[x]] \alpha * h[x] t[y],$ 
 $\quad \quad D[\sigma, h[x]] \beta * t[y],$ 
 $\quad \quad \gamma * h[x],$ 
 $\quad \quad ((\omega + \alpha) \delta - \gamma * \beta) / \omega$ 
 $\quad ]] // \beta\text{Collect}$ 
 $\quad];$ 
 $dm[x_, y_, z_][\beta_] := \beta // swapth[x, y] // hm[x, y, z] // tm[x, y, z];$ 
 $B /: B[w1_, \sigma1_, \mu1_] B[w2_, \sigma2_, \mu2_] := B[w1 * w2, \sigma1 + \sigma2, \omega2 \mu1 + \omega1 \mu2];$ 
 $(* Cheat *)$ 
 $Rp[x_, y_] := B[1, h[x] + T * h[y], (T - 1) * t[x] h[y]];$ 
 $Rm[x_, y_] := B[1, h[x] + h[y] / T, (1 / T - 1) * t[x] h[y]];$ 
 $(* End cheat *) Rp[x_, y_] := B[1, h[x] + T_x h[y], (T_x - 1) * t[x] h[y]];$ 
 $Rm[x_, y_] := B[1, h[x] + h[y] / T_x, (1 / T_x - 1) * t[x] h[y]];$ 
 $\text{Unprotect}[\text{NonCommutativeMultiply}];$ 
 $\text{Format}[\beta_B, \text{StandardForm}] := \beta\text{Form}[\beta];$ 

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{Rp[1, 2], Rm[1, 2]}

$$\left\{ \begin{pmatrix} 1 & h[1] & h[2] \\ t[1] & 0 & -1 + T_1 \\ 1 + \Sigma/\omega & 1 & T_1 \end{pmatrix}, \begin{pmatrix} 1 & h[1] & h[2] \\ t[1] & 0 & -\frac{-1+T_1}{T_1} \\ 1 + \Sigma/\omega & 1 & \frac{1}{T_1} \end{pmatrix} \right\}$$

Knot[8, 17] is calculated as in 120406 Calculator.nb.

Rm[12, 1]

$$\begin{pmatrix} 1 & h[1] & h[12] \\ t[12] & -\frac{-1+T_{12}}{T_{12}} & 0 \\ 1 + \Sigma/\omega & \frac{1}{T_{12}} & 1 \end{pmatrix}$$

Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]

$$\begin{pmatrix} 1 & h[1] & h[7] & h[12] \\ t[1] & 0 & -\frac{-1+T_1}{T_1} & 0 \\ t[12] & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 \\ 1 + \Sigma/\omega & \frac{1}{T_{12}} & \frac{1}{T_1} & 1 \end{pmatrix}$$

(Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]

$$\begin{pmatrix} 1 & h[1] & h[7] & h[8] & h[12] \\ t[1] & 0 & -\frac{-1+T_1}{T_1 T_8} & 0 & 0 \\ t[8] & -\frac{-1+T_8}{T_8 T_{12}} & -\frac{(-1+T_1)(-1+T_8)}{T_1 T_8} & 0 & 0 \\ t[12] & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 & 0 \\ 1 + \Sigma/\omega & \frac{1}{T_8 T_{12}} & \frac{1}{T_1} & 1 & 1 \end{pmatrix}$$

((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4, 1]

$$\begin{pmatrix} 1 & h[1] & h[7] & h[8] & h[11] & h[12] \\ t[1] & 0 & -\frac{-1+T_1}{T_1 T_8} & 0 & -\frac{-1+T_1}{T_1} & 0 \\ t[8] & -\frac{-1+T_8}{T_8 T_{12}} & -\frac{(-1+T_1)(-1+T_8)}{T_1 T_8} & 0 & 0 & 0 \\ t[12] & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 & 0 & 0 \\ 1 + \Sigma/\omega & \frac{1}{T_8 T_{12}} & \frac{1}{T_1} & 1 & \frac{1}{T_1} & 1 \end{pmatrix}$$

((((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4, 1])

Rp[16, 5] // dm[1, 5, 1]

$$\begin{pmatrix} 1 & h[1] & h[7] & h[8] & h[11] & h[12] & h[16] \\ t[1] & 0 & -\frac{(-1+T_1) T_{16}}{T_1 T_8} & 0 & -\frac{(-1+T_1) T_{16}}{T_1} & 0 & 0 \\ t[8] & -\frac{-1+T_8}{T_8 T_{12}} & -\frac{(-1+T_1)(-1+T_8)}{T_1 T_8} & 0 & 0 & 0 & 0 \\ t[12] & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 & 0 & 0 & 0 \\ t[16] & \frac{-1+T_{16}}{T_8 T_{12}} & \frac{(-1+T_1)(-1+T_{16})}{T_1 T_8} & 0 & \frac{(-1+T_1)(-1+T_{16})}{T_1} & 0 & 0 \\ 1 + \Sigma/\omega & \frac{T_{16}}{T_8 T_{12}} & \frac{1}{T_1} & 1 & \frac{1}{T_1} & 1 & 1 \end{pmatrix}$$

$$(((\text{Rm}[12, 1] \text{Rm}[2, 7] // \text{dm}[1, 2, 1]) \text{Rm}[8, 3] // \text{dm}[1, 3, 1]) \text{Rm}[4, 11] // \text{dm}[1, 4, 1]) \\ \text{Rp}[16, 5] // \text{dm}[1, 5, 1]) \text{Rp}[6, 13] // \text{dm}[1, 6, 1]$$

$$\left(\begin{array}{ccccccc} 1 & h[1] & h[7] & h[8] & h[11] & h[12] & h[13] & h[16] \\ t[1] & 0 & -\frac{(-1+T_1) T_{16}}{T_1 T_8} & 0 & -\frac{(-1+T_1) T_{16}}{T_1} & 0 & -1 + T_1 & 0 \\ t[8] & -\frac{-1+T_8}{T_8 T_{12}} & -\frac{(-1+T_1) (-1+T_8)}{T_1 T_8} & 0 & 0 & 0 & 0 & 0 \\ t[12] & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\ t[16] & \frac{-1+T_{16}}{T_8 T_{12}} & \frac{(-1+T_1) (-1+T_{16})}{T_1 T_8} & 0 & \frac{(-1+T_1) (-1+T_{16})}{T_1} & 0 & 0 & 0 \\ 1+\Sigma/\omega & \frac{T_{16}}{T_8 T_{12}} & \frac{1}{T_1} & 1 & \frac{1}{T_1} & 1 & T_1 & 1 \end{array} \right)$$

$$(((\text{Rm}[12, 1] \text{Rm}[2, 7] // \text{dm}[1, 2, 1]) \text{Rm}[8, 3] // \text{dm}[1, 3, 1]) \text{Rm}[4, 11] // \text{dm}[1, 4, 1]) \\ \text{Rp}[16, 5] // \text{dm}[1, 5, 1]) \text{Rp}[6, 13] // \text{dm}[1, 6, 1] // \text{dm}[1, 7, 1]$$

$$\left(\begin{array}{ccccccc} \frac{T_1 T_8 + T_{16} - T_1 T_{16}}{T_1 T_8} & h[1] & h[8] & h[11] & h[12] & h[13] & h[16] \\ t[1] & -\frac{(-1+T_1) T_{16}^2}{T_1^2 T_8^2 T_{12}} & 0 & -\frac{(-1+T_1) T_{16}}{T_1^2} & 0 & \frac{-1+T_1}{T_1} & 0 \\ t[8] & -\frac{-1+T_8}{T_8 T_{12}} & 0 & -\frac{(-1+T_1)^2 (-1+T_8) T_{16}}{T_1^2 T_8} & 0 & \frac{(-1+T_1)^2 (-1+T_8)}{T_1 T_8} & 0 \\ t[12] & -\frac{(-1+T_{12}) (T_1 T_8 + T_{16} - T_1 T_{16})}{T_1 T_8 T_{12}} & 0 & 0 & 0 & 0 & 0 \\ t[16] & \frac{-1+T_{16}}{T_8 T_{12}} & 0 & \frac{(-1+T_1) (-1+T_{16})}{T_1} & 0 & -\frac{(-1+T_1)^2 (-1+T_{16})}{T_1 T_8} & 0 \\ 1+\Sigma/\omega & \frac{T_{16}}{T_1 T_8 T_{12}} & 1 & \frac{1}{T_1} & 1 & T_1 & 1 \end{array} \right)$$

$$1 + \text{Total}[\{\{-1 + \frac{1}{T}\}, \{-\frac{(-1+T)^3}{T^2}\}, \{0\}, \{-2 + \frac{1}{T} + T\}\}] / (1/T) // \text{Simplify}$$

$$\left\{ \frac{1}{T} \right\}$$

$$(((\text{Rm}[12, 1] \text{Rm}[2, 7] // \text{dm}[1, 2, 1]) \text{Rm}[8, 3] // \text{dm}[1, 3, 1]) \text{Rm}[4, 11] // \text{dm}[1, 4, 1]) \\ \text{Rp}[16, 5] // \text{dm}[1, 5, 1]) \\ \text{Rp}[6, 13] // \text{dm}[1, 6, 1] // \text{dm}[1, 7, 1] // \text{dm}[11, 12, 11]$$

$$\left(\begin{array}{cccccc} \frac{T_1 T_8 + T_{16} - T_1 T_{16}}{T_1 T_8} & h[1] & h[8] & h[11] & h[13] & h[16] \\ t[1] & -\frac{(-1+T_1) T_{16}^2}{T_1^2 T_8^2 T_{11}} & 0 & -\frac{(-1+T_1) T_{16}}{T_1^2} & \frac{-1+T_1}{T_1} & 0 \\ t[8] & -\frac{-1+T_8}{T_8 T_{11}} & 0 & -\frac{(-1+T_1)^2 (-1+T_8) T_{16}}{T_1^2 T_8} & \frac{(-1+T_1)^2 (-1+T_8)}{T_1 T_8} & 0 \\ t[11] & -\frac{(-1+T_{11}) (T_1 T_8 + T_{16} - T_1 T_{16})}{T_1 T_8 T_{11}} & 0 & 0 & 0 & 0 \\ t[16] & \frac{-1+T_{16}}{T_8 T_{11}} & 0 & \frac{(-1+T_1) (-1+T_{16})}{T_1} & -\frac{(-1+T_1)^2 (-1+T_{16})}{T_1 T_8} & 0 \\ 1+\Sigma/\omega & \frac{T_{16}}{T_1 T_8 T_{11}} & 1 & \frac{1}{T_1} & T_1 & 1 \end{array} \right)$$

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t29 =
(((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4, 1])
Rp[16, 5] // dm[1, 5, 1]) Rp[6, 13] //
dm[1, 6, 1] // dm[1, 7, 1] // dm[11, 12, 11] // dm[11, 13, 11]


$$\left( \begin{array}{ccccc} \frac{T_1 T_8 + T_{16} - T_1 T_{16}}{T_1 T_8} & h[1] & h[8] & h[11] & h[16] \\ t[1] & \frac{(-1+T_1) (-T_1 T_8^2 + T_1 T_8^2 T_{11} - T_{16}^2)}{T_1^2 T_8^2 T_{11}} & 0 & -\frac{(-1+T_1) (-1+T_{16})}{T_1^2} & 0 \\ t[8] & \frac{(-1+T_8) (-1+T_1 - T_1^2 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11})}{T_1 T_8 T_{11}} & 0 & -\frac{(-1+T_1)^2 (-1+T_8) (-1+T_{16})}{T_1^2 T_8} & 0 \\ t[11] & -\frac{(-1+T_{11}) (T_1 T_8 + T_{16} - T_1 T_{16})}{T_8 T_{11}} & 0 & 0 & 0 \\ t[16] & -\frac{(-1+T_1 - T_1^2 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11}) (-1+T_{16})}{T_1 T_8 T_{11}} & 0 & \frac{(-1+T_1) (1-T_1 + T_1 T_8) (-1+T_{16})}{T_1^2 T_8} & 0 \\ 1+\Sigma/\omega & \frac{T_{16}}{T_1 T_8 T_{11}} & 1 & 1 & 1 \end{array} \right)$$


Collect[t29[[3]] /. t[_] → 1, _h, Expand]

h[1]  $\left( -1 + \frac{T_{16}}{T_8} - \frac{T_{16}}{T_1 T_8} + \frac{T_{16}}{T_1 T_8 T_{11}} + \frac{T_{16}^2}{T_1^2 T_8^2 T_{11}} - \frac{T_{16}^2}{T_1 T_8^2 T_{11}} \right)$ 

(((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4, 1])
Rp[16, 5] // dm[1, 5, 1]) Rp[6, 13] // dm[1, 6, 1] //
dm[1, 7, 1] // dm[11, 12, 11] // dm[11, 13, 11] // dm[1, 8, 1]


$$\left( \begin{array}{ccccc} \frac{T_1^2 + T_{16} - T_1 T_{16}}{T_1^2} & h[1] & h[11] & h[16] \\ t[1] & \frac{(-1+T_1) (-T_1^2 - T_1^4 + T_1^2 T_{11} - T_1^3 T_{11} + T_1^4 T_{11} - T_{16}^2)}{T_1^4 T_{11}} & -\frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{16})}{T_1^3} & 0 \\ t[11] & -\frac{(-1+T_{11}) (T_1^2 + T_{16} - T_1 T_{16})}{T_1 T_{11}} & 0 & 0 \\ t[16] & -\frac{(-1+T_1 - T_1^2 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11}) (-1+T_{16})}{T_1^2 T_{11}} & \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{16})}{T_1^3} & 0 \\ 1+\Sigma/\omega & \frac{T_{16}}{T_1^2 T_{11}} & 1 & 1 \end{array} \right)$$


(((((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4, 1])
Rp[16, 5] // dm[1, 5, 1]) Rp[6, 13] // dm[1, 6, 1] // dm[1, 7, 1] //
dm[11, 12, 11] // dm[11, 13, 11] // dm[1, 8, 1]) Rp[14, 9]


$$\left( \begin{array}{ccccccc} \frac{T_1^2 + T_{16} - T_1 T_{16}}{T_1^2} & h[1] & h[9] & h[11] & h[14] & h[16] \\ t[1] & \frac{(-1+T_1) (-T_1^2 - T_1^4 + T_1^2 T_{11} - T_1^3 T_{11} + T_1^4 T_{11} - T_{16}^2)}{T_1^4 T_{11}} & 0 & -\frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{16})}{T_1^3} & 0 & 0 \\ t[11] & -\frac{(-1+T_{11}) (T_1^2 + T_{16} - T_1 T_{16})}{T_1 T_{11}} & 0 & 0 & 0 & 0 \\ t[14] & 0 & \frac{(-1+T_{14}) (T_1^2 + T_{16} - T_1 T_{16})}{T_1^2} & 0 & 0 & 0 \\ t[16] & -\frac{(-1+T_1 - T_1^2 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11}) (-1+T_{16})}{T_1^2 T_{11}} & 0 & \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{16})}{T_1^3} & 0 & 0 \\ 1+\Sigma/\omega & \frac{T_{16}}{T_1^2 T_{11}} & T_{14} & 1 & 1 & 1 \end{array} \right)$$


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t33 =

$$\begin{aligned} & (((((\text{Rm}[12, 1] \text{Rm}[2, 7] // \text{dm}[1, 2, 1]) \text{Rm}[8, 3] // \text{dm}[1, 3, 1]) \text{Rm}[4, 11] // \text{dm}[1, 4, 1]) \\ & \quad \text{Rp}[16, 5] // \text{dm}[1, 5, 1]) \text{Rp}[6, 13] // \text{dm}[1, 6, 1] // \\ & \quad \text{dm}[1, 7, 1] // \text{dm}[11, 12, 11] // \text{dm}[11, 13, 11] // \text{dm}[1, 8, 1]) \\ & \quad \text{Rp}[14, 9] // \text{dm}[11, 14, 11] // \text{dm}[1, 9, 1]) \end{aligned}$$

$$\left\{ \begin{array}{lll} \frac{T_1^2 + T_{16} - T_1 T_{16}}{T_1^2} & h[1] & h[11] \quad h[16] \\ t[1] & \frac{(-1+T_1) (-T_1^2 - T_1^4 + T_1^2 T_{11} - T_1^3 T_{11} + T_1^4 T_{11} - T_{16}^2)}{T_1^4} & - \frac{(-1+T_1) (1-T_1 + T_1^2) T_{11} (-1+T_{16})}{T_1^3} \quad 0 \\ t[11] & - \frac{(1-T_1 + T_1^2) (-1+T_{11}) (1-T_{11} + T_1 T_{11} - T_{16})}{T_1^2 T_{11}} & \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{11}) (-1+T_{16})}{T_1^3} \quad 0 \\ t[16] & - \frac{(-1+T_1 - T_1^2 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11}) (-1+T_{16})}{T_1^2 T_{11}} & \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{16})}{T_1^3} \quad 0 \\ 1 + \Sigma/\omega & \frac{T_{16}}{T_1^2} & 1 \quad 1 \end{array} \right\}$$

Collect[t33[[3]] /. t[_] → 1, _h, Expand]

$$\begin{aligned} & h[1] \left(-1 + \frac{T_{16}}{T_1} + \frac{T_{16}^2}{T_1^4} - \frac{T_{16}^2}{T_1^3} \right) \\ & (((((\text{Rm}[12, 1] \text{Rm}[2, 7] // \text{dm}[1, 2, 1]) \text{Rm}[8, 3] // \text{dm}[1, 3, 1]) \text{Rm}[4, 11] // \text{dm}[1, 4, 1]) \\ & \quad \text{Rp}[16, 5] // \text{dm}[1, 5, 1]) \text{Rp}[6, 13] // \text{dm}[1, 6, 1] // \\ & \quad \text{dm}[1, 7, 1] // \text{dm}[11, 12, 11] // \text{dm}[11, 13, 11] // \text{dm}[1, 8, 1]) \\ & \quad \text{Rp}[14, 9] // \text{dm}[11, 14, 11] // \text{dm}[1, 9, 1]) \text{Rp}[10, 15] \end{aligned}$$

$$\left\{ \begin{array}{lll} \frac{T_1^2 + T_{16} - T_1 T_{16}}{T_1^2} & h[1] & h[10] \quad h[11] \quad h[15] \\ t[1] & \frac{(-1+T_1) (-T_1^2 - T_1^4 + T_1^2 T_{11} - T_1^3 T_{11} + T_1^4 T_{11} - T_{16}^2)}{T_1^4} & 0 \quad - \frac{(-1+T_1) (1-T_1 + T_1^2) T_{11} (-1+T_{16})}{T_1^3} \quad 0 \\ t[10] & 0 & 0 \quad 0 \quad \frac{(-1+T_{10}) (T_1^2 + T_{16} - T_1 T_{16})}{T_1^2} \\ t[11] & - \frac{(1-T_1 + T_1^2) (-1+T_{11}) (1-T_{11} + T_1 T_{11} - T_{16})}{T_1^2 T_{11}} & 0 \quad \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{11}) (-1+T_{16})}{T_1^3} \quad 0 \\ t[16] & - \frac{(-1+T_1 - T_1^2 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11}) (-1+T_{16})}{T_1^2 T_{11}} & 0 \quad \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{16})}{T_1^3} \quad 0 \\ 1 + \Sigma/\omega & \frac{T_{16}}{T_1^2} & 1 \quad 1 \quad T_{10} \end{array} \right\}$$

$$\begin{aligned} & (((((\text{Rm}[12, 1] \text{Rm}[2, 7] // \text{dm}[1, 2, 1]) \text{Rm}[8, 3] // \text{dm}[1, 3, 1]) \text{Rm}[4, 11] // \text{dm}[1, 4, 1]) \\ & \quad \text{Rp}[16, 5] // \text{dm}[1, 5, 1]) \text{Rp}[6, 13] // \text{dm}[1, 6, 1] // \text{dm}[1, 7, 1] // \\ & \quad \text{dm}[11, 12, 11] // \text{dm}[11, 13, 11] // \text{dm}[1, 8, 1]) \text{Rp}[14, 9] // \\ & \quad \text{dm}[11, 14, 11] // \text{dm}[1, 9, 1]) \text{Rp}[10, 15] // \text{dm}[1, 10, 1] \end{aligned}$$

$$\left\{ \begin{array}{lll} \frac{T_1^2 + T_{16} - T_1 T_{16}}{T_1^2} & h[1] & h[11] \quad h[15] \quad h[16] \\ t[1] & \frac{(-1+T_1) (-T_1^2 - T_1^4 + T_1^2 T_{11} - T_1^3 T_{11} + T_1^4 T_{11} - T_{16}^2)}{T_1^4} & - \frac{(-1+T_1) (1-T_1 + T_1^2) T_{11} (-1+T_{16})}{T_1^3} \quad \frac{(-1+T_1) (T_1^2 + T_{16} - T_1 T_{16})}{T_1^2} \quad 0 \\ t[11] & - \frac{(1-T_1 + T_1^2) (-1+T_{11}) (1-T_{11} + T_1 T_{11} - T_{16})}{T_1^2 T_{11}} & \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{11}) (-1+T_{16})}{T_1^3} \quad 0 \\ t[16] & - \frac{(-1+T_1 - T_1^2 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11}) (-1+T_{16})}{T_1^2 T_{11}} & \frac{(-1+T_1) (1-T_1 + T_1^2) (-1+T_{16})}{T_1^3} \quad 0 \\ 1 + \Sigma/\omega & \frac{T_{16}}{T_1^2} & 1 \quad T_1 \quad 1 \end{array} \right\}$$

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((((((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4, 1])
    Rp[16, 5] // dm[1, 5, 1]) Rp[6, 13] // dm[1, 6, 1] // dm[1, 7, 1] //
dm[11, 12, 11] // dm[11, 13, 11] // dm[1, 8, 1]) Rp[14, 9] //
dm[11, 14, 11] // dm[1, 9, 1]) Rp[10, 15] // dm[1, 10, 1] // dm[11, 15, 11]


$$\left( \begin{array}{l} \frac{T_1^2 + T_{16} - T_1 T_{16}}{T_1^2} \\ t[1] \\ t[11] \\ t[16] \\ 1 + \Sigma/\omega \end{array} \right) \left( \begin{array}{l} h[1] \\ \frac{(-1+T_1) (-T_1^2 + T_1^3 - T_1^4 + T_1^2 T_{11} - 3 T_1^3 T_{11} + 2 T_1^4 T_{11} - T_1^5 T_{11} + T_1^3 T_{11}^2 - T_1^4 T_{11}^2 + T_1^5 T_{11}^2 + T_1^2 T_{16} - T_1^3 T_{16} + T_1^4 T_{16} - T_1^2 T_{11} T_{16} + T_1^3 T_{11} T_{16} - T_1^4 T_{11} T_{16})}{T_1^4 T_{11}} \\ - \frac{(1-T_1+T_1^2) (-1+T_{11}) (1-T_{11}+T_1 T_{11}-T_{16})}{T_1 T_{11}} \\ - \frac{(-1+T_1-T_1^2+T_{11}-2 T_1 T_{11}+T_1^2 T_{11}) (-1+T_{16})}{T_1^2 T_{11}} \\ \frac{T_{16}}{T_1^2} \end{array} \right)$$


((((((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4, 1])
    Rp[16, 5] // dm[1, 5, 1]) Rp[6, 13] // dm[1, 6, 1] //
dm[1, 7, 1] // dm[11, 12, 11] // dm[11, 13, 11] // dm[1, 8, 1])
Rp[14, 9] // dm[11, 14, 11] // dm[1, 9, 1]) Rp[10, 15] //
dm[1, 10, 1] // dm[11, 15, 11] // dm[11, 16, 11]


$$\left( \begin{array}{l} \frac{T_1^2 + T_{11} - T_1 T_{11}}{T_1^2} \\ t[1] \\ t[11] \\ 1 + \Sigma/\omega \end{array} \right) \left( \begin{array}{l} h[1] \\ \frac{(-1+T_1) (-T_1^2 + T_1^3 - T_1^4 + 2 T_1^2 T_{11} - 4 T_1^3 T_{11} + 3 T_1^4 T_{11} - T_1^5 T_{11} - T_1^2 T_{11}^2 + 2 T_1^3 T_{11}^2 - 2 T_1^4 T_{11}^2 + T_1^5 T_{11}^2 - T_1^3 T_{11})}{T_1^4 T_{11}} \\ - \frac{(-1+T_{11}) (-1+2 T_1 - 2 T_1^2 + T_{11} - 4 T_1 T_{11} + 4 T_1^2 T_{11} - 3 T_1^3 T_{11} + T_1^4 T_{11})}{T_1^2 T_{11}} \\ \frac{T_{11}}{T_1^2} \end{array} \right) \frac{(-1+T_1) (1-T_1+T_1^2)}{(-1+T_1)}$$


t40 =
((((((Rm[12, 1] Rm[2, 7] // dm[1, 2, 1]) Rm[8, 3] // dm[1, 3, 1]) Rm[4, 11] // dm[1, 4,
1]) Rp[16, 5] // dm[1, 5, 1]) Rp[6, 13] // dm[1, 6, 1] // dm[
1, 7, 1] // dm[11, 12, 11] // dm[11, 13, 11] // dm[1, 8, 1])
Rp[14, 9] // dm[11, 14, 11] // dm[1, 9, 1]) Rp[10, 15] //
dm[1, 10, 1] // dm[11, 15, 11] // dm[11, 16, 11] // dm[1, 11, 1]


$$\left( \begin{array}{l} - \frac{1-4 T_1+8 T_1^2-11 T_1^3+8 T_1^4-4 T_1^5+T_1^6}{T_1^3} h[1] \\ t[1] \\ 1 + \Sigma/\omega \end{array} \right) \left( \begin{array}{l} 0 \\ 1 \end{array} \right)$$


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