Pensieve Header: Meta-group calculus in the Burau presentation; continues pensieve://2012-03/.

```
<< KnotTheory`
GD[K_] := GD @@ (
   PD[K] /. X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]],
      Ar[l, i, +1], Ar[j, i, -1]
     ]
  )
```

```
Loading KnotTheory` version of August 22, 2010, 13:36:57.55.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
BuSimplify = Simplify;
SetAttributes[BuCollect, Listable];
BuCollect[Bu[ω_, μ_]] := Bu[
   BuSimplify[ω],
   Collect[μ, _h, Collect[#, _t, BuSimplify] &]
  ];
L[β_] := Union[Cases[β, h[s_] | t[s_] | T_s_ :> s, Infinity]];
BuForm[Bu[ω_, μ_]] := Module[
   {labels, mat},
   labels = L[Bu[ω, μ]];
   mat = Outer[BuSimplify[Coefficient[μ, h[#1] t[#2]]] &, labels, labels];
   PrependTo[mat, t /@ labels];
   mat = Prepend[Transpose[mat], Prepend[h /@ labels, ω]];
   MatrixForm[mat]
  ];
BuForm[else_] := else /. β_Bu :> BuForm[β];
```

```
⟨α_⟩ := α /. t[_] → 1;
m[x_, y_, z_][Bu[ω_, μ_]] := Module[
   {α, β, γ, δ, ϵ, η, Θ},
   α = Coefficient[μ, t[x] h[y]];
   β = Coefficient[μ, t[x]] /. h[y] → 0;
   γ = Coefficient[μ, t[y] h[x]];
   δ = Coefficient[μ, t[y]] /. h[x] → 0;
   ϵ = Coefficient[μ, h[x]] /. t[y] → 0;
   η = Coefficient[μ, h[y]] /. t[x] → 0;
   Θ = μ /. t[x] | t[y] | h[x] | h[y] → 0;
   (Bu[ω (1 - α),
       t[z] (δ + β (1 + ⟨η⟩ / (α - 1))) + h[z] (ϵ + η (γ + ⟨ϵ⟩) / (α - 1))
        + Θ - (Expand[η * β] /. t[s_] h[s_] → 0) / (α - 1)
      ] /. T_x | T_y → T_z) // BuCollect
  ];
Bu /: Bu[ω1_, μ1_] * Bu[ω2_, μ2_] := Bu[ω1 * ω2, μ1 + μ2];
Rp[x_, y_] := Bu[1, t[x] h[y] (1 - T_x)];
Rm[x_, y_] := Bu[1, t[x] h[y] (1 - 1 / T_x)];
```

```
{β = Times @@
    (GD[Knot[8, 17]] /. {Ar[x_, y_, 1] :> Rp[x, y], Ar[x_, y_, -1] :> Rm[x, y]}),
  β // BuForm,
  Do[β = m[1, k, 1][β], {k, 2, 16}]; β // BuForm,
  Alexander[Knot[8, 17]][T1]
 } // ColumnForm
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\text{Bu}\left[1,\ h[6]\ (1-T_1)\ t[1]+h[8]\left(1-\frac{1}{T_3}\right)t[3]+h[12]\left(1-\frac{1}{T_5}\right)t[5]+h[14]\ (1-T_7)\ t[7]+h[4]\left(1-\frac{1}{T_9}\right.\right.$$

	1	h[1]	h[2]	h[3]	h[4]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]	h[11]	h[12]	h[13]	h[1
t[1]	0	0	0	0	0	$1-T_1$	0	0	0	0	0	0	0	0	
t[2]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[3]	0	0	0	0	0	0	0	$1-\frac{1}{T_3}$	0	0	0	0	0	0	
t[4]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[5]	0	0	0	0	0	0	0	0	0	0	0	$1-\frac{1}{T_5}$	0	0	
t[6]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[7]	0	0	0	0	0	0	0	0	0	0	0	0	0	$1-$	
t[8]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[9]	0	0	0	$1-\frac{1}{T_9}$	0	0	0	0	0	0	0	0	0	0	
t[10]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[11]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[12]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[13]	0	$1-\frac{1}{T_{13}}$	0	0	0	0	0	0	0	0	0	0	0	0	
t[14]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
t[15]	0	0	0	0	0	0	0	0	0	$1-T_{15}$	0	0	0	0	
t[16]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

$$\begin{pmatrix} -8-\frac{1}{T_1^2}+\frac{4}{T_1}+11\,T_1-8\,T_1^2+4\,T_1^3-T_1^4 & h[1] \\ t[1] & 0 \end{pmatrix}$$

$$11-\frac{1}{T_1^3}+\frac{4}{T_1^2}-\frac{8}{T_1}-8\,T_1+4\,T_1^2-T_1^3$$

```
BuZ[K_] := Module[{β},
   β = Times @@ (GD[K] /. {Ar[x_, y_, 1] :> Rp[x, y], Ar[x_, y_, -1] :> Rm[x, y]});
   Do[β = m[1, k, 1][β], {k, 2, 2 Crossings[K]}];
   β[[1]]
  ];
BuZ[Knot[8, 17]]
```

$$-8-\frac{1}{T_1^2}+\frac{4}{T_1}+11\,T_1-8\,T_1^2+4\,T_1^3-T_1^4$$

```
Factor[Alexander[#][T1] / BuZ[#]] & /@ AllKnots[{3, 7}]
```

$$\left\{T_1,\ \frac{1}{T_1},\ T_1^2,\ T_1^2,\ 1,\ 1,\ 1,\ T_1^3,\ T_1^3,\ \frac{1}{T_1^4},\ \frac{1}{T_1^4},\ T_1^3,\ T_1,\ \frac{1}{T_1^2}\right\}$$

```
n = 5;
{
  β1 = Bu[ω,
    Sum[α10 i+j t[i] h[j] - α11 j / n * t[j] h[j], {i, n}, {j, n}]
   ],
  β1 // m[1, 2, 1]
 } // BuForm // ColumnForm
```

	ω	h[1]	h[2]	h[3]	h[4]	h[5]
t[1]	0	α_{12}	α_{13}	α_{14}	α_{15}
t[2]	α_{21}	0	α_{23}	α_{24}	α_{25}
t[3]	α_{31}	α_{32}	0	α_{34}	α_{35}
t[4]	α_{41}	α_{42}	α_{43}	0	α_{45}
t[5]	α_{51}	α_{52}	α_{53}	α_{54}	0

	$\omega - \omega\,\alpha_{12}$	h[1]	h[3]	h[4]	h[5]
t[1]	0	$\alpha_{23} + \alpha_{13}\left(1 + \frac{\alpha_{32}+\alpha_{42}+\alpha_{52}}{-1+\alpha_{12}}\right)$	$\alpha_{24} + \alpha_{14}\left(1 + \frac{\alpha_{32}+\alpha_{42}+\alpha_{52}}{-1+\alpha_{12}}\right)$	$\alpha_{25} + \alpha_{15}\left(1 + \frac{\alpha_{32}+\alpha_{42}+$
t[3]	$\alpha_{31} + \frac{\alpha_{32}\,(\alpha_{21}+\alpha_{31}+\alpha_{41}+\alpha_{51})}{-1+\alpha_{12}}$	0	$-\frac{\alpha_{14}\,\alpha_{32}}{-1+\alpha_{12}} + \alpha_{34}$	$-\frac{\alpha_{15}\,\alpha_{32}}{-1+\alpha_{12}} + \alpha_{35}$
t[4]	$\alpha_{41} + \frac{\alpha_{42}\,(\alpha_{21}+\alpha_{31}+\alpha_{41}+\alpha_{51})}{-1+\alpha_{12}}$	$-\frac{\alpha_{13}\,\alpha_{42}}{-1+\alpha_{12}} + \alpha_{43}$	0	$-\frac{\alpha_{15}\,\alpha_{42}}{-1+\alpha_{12}} + \alpha_{45}$
t[5]	$\alpha_{51} + \frac{(\alpha_{21}+\alpha_{31}+\alpha_{41}+\alpha_{51})\,\alpha_{52}}{-1+\alpha_{12}}$	$-\frac{\alpha_{13}\,\alpha_{52}}{-1+\alpha_{12}} + \alpha_{53}$	$-\frac{\alpha_{14}\,\alpha_{52}}{-1+\alpha_{12}} + \alpha_{54}$	0

```
n = 5;
{
  β1 = Bu[ω,
    Sum[α10 i+j t[i] h[j] - α11 j / n * t[j] h[j], {i, n}, {j, n}]
   ],
  β2 =
   (((β1 /. Bu[ω_, μ_] :↦ Bu[ω, μ / ω]) // m[1, 2, 1]) /. Bu[ω_, μ_] :↦ Bu[ω, μ * ω]),
  Collect[⟨β2[[2]]⟩, _h, FullSimplify],
  Collect[β2[[2]] /. _h → 1, _t, FullSimplify]
 } // BuForm // ColumnForm
```

	ω	h[1]	h[2]	h[3]	h[4]	h[5]
t[1]	0	α_{12}	α_{13}	α_{14}	α_{15}
t[2]	α_{21}	0	α_{23}	α_{24}	α_{25}
t[3]	α_{31}	α_{32}	0	α_{34}	α_{35}
t[4]	α_{41}	α_{42}	α_{43}	0	α_{45}
t[5]	α_{51}	α_{52}	α_{53}	α_{54}	0

	$\omega - \alpha_{12}$	h[1]	h[3]	h[4]	h[5]
t[1]	0	$\frac{(\omega-\alpha_{12})\left(\alpha_{23}+\alpha_{13}\left(1+\frac{\alpha_{32}+\alpha_{42}+\alpha_{52}}{-\omega+\alpha_{12}}\right)\right)}{\omega}$	$\frac{(\omega-\alpha_{12})\left(\alpha_{24}+\alpha_{14}\left(1+\frac{\alpha_{32}+\alpha_{42}+\alpha_{52}}{-\omega+\alpha_{12}}\right)\right)}{\omega}$	$\frac{(\omega-\alpha_{12})\left(\alpha_{25}+\alpha_{15}\left(1+$
t[3]	$\frac{\alpha_{31}\,(\omega-\alpha_{12}-\alpha_{32})-\alpha_{32}\,(\alpha_{21}+\alpha_{41}+\alpha_{51})}{\omega}$	0	$\frac{\alpha_{14}\,\alpha_{32}+(\omega-\alpha_{12})\,\alpha_{34}}{\omega}$	$\frac{\alpha_{15}\,\alpha_{32}+(\omega-\alpha_{12}$
t[4]	$\frac{\alpha_{41}\,(\omega-\alpha_{12}-\alpha_{42})-\alpha_{42}\,(\alpha_{21}+\alpha_{31}+\alpha_{51})}{\omega}$	$\frac{\alpha_{13}\,\alpha_{42}+(\omega-\alpha_{12})\,\alpha_{43}}{\omega}$	0	$\frac{\alpha_{15}\,\alpha_{42}+(\omega-\alpha_{12}$
t[5]	$\frac{\alpha_{51}\,(\omega-\alpha_{12}-\alpha_{52})-(\alpha_{21}+\alpha_{31}+\alpha_{41})\,\alpha_{52}}{\omega}$	$\frac{\alpha_{13}\,\alpha_{52}+(\omega-\alpha_{12})\,\alpha_{53}}{\omega}$	$\frac{\alpha_{14}\,\alpha_{52}+(\omega-\alpha_{12})\,\alpha_{54}}{\omega}$	0

$$\frac{h[5]\,((\omega-\alpha_{12})\,(\alpha_{15}+\alpha_{25}+\alpha_{35}+\alpha_{45})-\alpha_{15}\,\alpha_{52})}{\omega} + \frac{h[1]\,((\alpha_{31}+\alpha_{41}+\alpha_{51})\,(\omega-\alpha_{12}-\alpha_{32}-\alpha_{42}-\alpha_{52})-\alpha_{21}\,(\alpha_{32}+\alpha_{42}+\alpha_{52}))}{\omega} + \frac{h[3]\,(\alpha_{13}\,(\omega-\alpha_{12}-\alpha_{32})+(\omega-\alpha_{12})\,($$

$$\frac{(\omega-\alpha_{12})\left(\alpha_{14}+\alpha_{15}+\alpha_{23}+\alpha_{24}+\alpha_{25}+\frac{\alpha_{13}\,(\omega-\alpha_{12}-\alpha_{32}-\alpha_{42}-\alpha_{52})-(\alpha_{14}+\alpha_{15})\,(\alpha_{32}+\alpha_{42}+\alpha_{52})}{\omega-\alpha_{12}}\right)\,t[1]}{\omega} + \frac{(\alpha_{31}\,(\omega-\alpha_{12}-\alpha_{32})+(\omega-\alpha_{12})\,(\alpha_{34}+\alpha_{35})+\alpha_{32}\,(\alpha_{14}+\alpha_{15}-\alpha_{21}-\alpha_{41}-\alpha_{51}))\,t$$