```
SetDirectory["C:/drorbn/AcademicPensieve/2012-03/GWU_Talk/"]
```

```
C:\drorbn\AcademicPensieve\2012-03\GWU_Talk
```

Initialization

```
βSimp = Factor; SetAttributes[βCollect, Listable];
βCollect[B[ω_, Λ_]] := B[βSimp[ω],
   Collect[Λ, h_, Collect[#, t_, βSimp] &]];
βForm[B[ω_, Λ_]] := Module[{ts, hs, M},
   ts = Union[Cases[B[ω, Λ], (t | T)_s_ :> s, Infinity]];
   hs = Union[Cases[B[ω, Λ], h_s_ :> s, Infinity]];
   M = Outer[βSimp[Coefficient[Λ, h_#1 t_#2]] &, hs, ts];
   PrependTo[M, t_# & /@ ts];
   M = Prepend[Transpose[M], Prepend[h_# & /@ hs, ω]];
   MatrixForm[M]];
βForm[else_] := else /. β_B :> βForm[β];
Format[β_B, StandardForm] := βForm[β];
```

GetUtilities

```
<< Utilities.m
```

Program

```
⟨μ_⟩ := μ /. t_ → 1;
tm_{x_,y_→z_}[β_] := β /. {t_{x|y} → t_z, T_{x|y} → T_z};
hm_{x_,y_→z_}[B[ω_, Λ_]] := Module[
   {α = D[Λ, h_x], β = D[Λ, h_y], γ = Λ /. h_{x|y} → 0},
   B[ω, (α + (1 + ⟨α⟩) β) h_z + γ] // βCollect];
sw_{x_,y_}[B[ω_, Λ_]] := Module[{α, β, γ, δ, ϵ},
   α = Coefficient[Λ, h_y t_x]; β = D[Λ, t_x] /. h_y → 0;
   γ = D[Λ, h_y] /. t_x → 0;   δ = Λ /. h_y | t_x → 0;
   ϵ = 1 + α;
   B[ω * ϵ, α (1 + ⟨γ⟩ / ϵ) h_y t_x + β (1 + ⟨γ⟩ / ϵ) t_x
         + γ / ϵ h_y              + δ - γ * β / ϵ
    ] // βCollect];
gm_{x_,y_→z_}[β_] := β // sw_{x,y} // hm_{x,y→z} // tm_{x,y→z};
B /: B[ω1_, Λ1_] B[ω2_, Λ2_] := B[ω1 * ω2, Λ1 + Λ2];
Rp_{x_,y_} := B[1, (T_x - 1) t_x h_y];
Rm_{x_,y_} := B[1, (T_x^-1 - 1) t_x h_y];
```

htt

```
{β = B[ω, Sum[α10 i+j ti hj, {i, {1, 2, 3}}, {j, {4, 5}}]],
  β // tm1,2→1 // sw1,4,
  β // sw2,4 // sw1,4 // tm1,2→1
 } // ColumnForm
```

htt

$$\begin{pmatrix} \omega & h_4 & h_5 \\ t_1 & \alpha_{14} & \alpha_{15} \\ t_2 & \alpha_{24} & \alpha_{25} \\ t_3 & \alpha_{34} & \alpha_{35} \end{pmatrix}$$

$$\begin{pmatrix} \omega\ (1+\alpha_{14}+\alpha_{24}) & h_4 & h_5 \\ t_1 & \frac{(\alpha_{14}+\alpha_{24})\ (1+\alpha_{14}+\alpha_{24}+\alpha_{34})}{1+\alpha_{14}+\alpha_{24}} & \frac{(\alpha_{15}+\alpha_{25})\ (1+\alpha_{14}+\alpha_{24}+\alpha_{34})}{1+\alpha_{14}+\alpha_{24}} \\ t_3 & \frac{\alpha_{34}}{1+\alpha_{14}+\alpha_{24}} & \frac{-\alpha_{15}\ \alpha_{34}-\alpha_{25}\ \alpha_{34}+\alpha_{35}+\alpha_{14}\ \alpha_{35}+\alpha_{24}\ \alpha_{35}}{1+\alpha_{14}+\alpha_{24}} \end{pmatrix}$$

$$\begin{pmatrix} \omega\ (1+\alpha_{14}+\alpha_{24}) & h_4 & h_5 \\ t_1 & \frac{(\alpha_{14}+\alpha_{24})\ (1+\alpha_{14}+\alpha_{24}+\alpha_{34})}{1+\alpha_{14}+\alpha_{24}} & \frac{(\alpha_{15}+\alpha_{25})\ (1+\alpha_{14}+\alpha_{24}+\alpha_{34})}{1+\alpha_{14}+\alpha_{24}} \\ t_3 & \frac{\alpha_{34}}{1+\alpha_{14}+\alpha_{24}} & \frac{-\alpha_{15}\ \alpha_{34}-\alpha_{25}\ \alpha_{34}+\alpha_{35}+\alpha_{14}\ \alpha_{35}+\alpha_{24}\ \alpha_{35}}{1+\alpha_{14}+\alpha_{24}} \end{pmatrix}$$

R3

```
{Rm5,1 Rm6,2 Rp3,4 // gm1,4→1 // gm2,5→2 // gm3,6→3,
 Rp6,1 Rm2,4 Rm3,5 // gm1,4→1 // gm2,5→2 // gm3,6→3}
```

R3

$$\left\{\begin{pmatrix} 1 & h_1 & h_2 \\ t_2 & -\frac{-1+T_2}{T_2} & 0 \\ t_3 & \frac{-1+T_3}{T_2} & -\frac{-1+T_3}{T_3} \end{pmatrix}, \begin{pmatrix} 1 & h_1 & h_2 \\ t_2 & -\frac{-1+T_2}{T_2} & 0 \\ t_3 & \frac{-1+T_3}{T_2} & -\frac{-1+T_3}{T_3} \end{pmatrix}\right\}$$

8_17-1

```
β = Rm12,1 Rm2,7 Rm8,3 Rm4,11 Rp16,5 Rp6,13 Rp14,9 Rp10,15
```

8_17-1

$$\begin{pmatrix} 1 & h_1 & h_3 & h_5 & h_7 & h_9 & h_{11} & h_{13} & h_{15} \\ t_2 & 0 & 0 & 0 & -\frac{-1+T_2}{T_2} & 0 & 0 & 0 & 0 \\ t_4 & 0 & 0 & 0 & 0 & 0 & -\frac{-1+T_4}{T_4} & 0 & 0 \\ t_6 & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_6 & 0 \\ t_8 & 0 & -\frac{-1+T_8}{T_8} & 0 & 0 & 0 & 0 & 0 & 0 \\ t_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_{10} \\ t_{12} & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t_{14} & 0 & 0 & 0 & 0 & -1+T_{14} & 0 & 0 & 0 \\ t_{16} & 0 & 0 & -1+T_{16} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

8_17-2

```
Do[β = β // gm1,k→1, {k, 2, 10}]; β
```

8_17-2

$$\begin{pmatrix} \frac{T_1^2+T_{16}-T_1\ T_{16}}{T_1^2} & h_1 & h_{11} & h_{13} & h_{15} \\ t_1 & -\frac{(-1+T_1)\ T_{14}\ \left(T_1^3+T_{16}^2\right)}{T_1^2\ T_{12}\ \left(T_1^2+T_{16}-T_1\ T_{16}\right)} & -\frac{(-1+T_1)\ \left(1-T_1+T_1^2\right)\ T_{14}\ T_{16}}{T_1\ \left(T_1^2+T_{16}-T_1\ T_{16}\right)} & \frac{(-1+T_1)\ \left(1-T_1+T_1^2\right)\ T_{14}}{T_1^2+T_{16}-T_1\ T_{16}} & -1+T_1 \\ t_{12} & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 & 0 \\ t_{14} & \frac{(-1+T_{14})\ \left(-T_1+T_1^2+T_{16}\right)}{T_{12}\ \left(T_1^2+T_{16}-T_1\ T_{16}\right)} & \frac{(-1+T_1)\ \left(1-T_1+T_1^2\right)\ (-1+T_{14})\ T_{16}}{T_1\ \left(T_1^2+T_{16}-T_1\ T_{16}\right)} & -\frac{(-1+T_1)\ \left(1-T_1+T_1^2\right)\ (-1+T_{14})}{T_1^2+T_{16}-T_1\ T_{16}} & 0 \\ t_{16} & \frac{T_1\ (-1+T_{16})}{T_{12}\ \left(T_1^2+T_{16}-T_1\ T_{16}\right)} & \frac{(-1+T_1)\ T_1\ (-1+T_{16})}{T_1^2+T_{16}-T_1\ T_{16}} & -\frac{(-1+T_1)^2\ (-1+T_{16})}{T_1^2+T_{16}-T_1\ T_{16}} & 0 \end{pmatrix}$$

8_17-3

```
Do[β = β // gm_{1,k→1}, {k, 11, 16}]; β
```

8_17-3

$$\begin{pmatrix} -\frac{1-4\,T_1+8\,T_1^2-11\,T_1^3+8\,T_1^4-4\,T_1^5+T_1^6}{T_1^3} & h_1 \\ t_1 & 0 \end{pmatrix}$$

8_17-4

```
<< KnotTheory`
Alexander[Knot[8, 17]][T_1] // Factor
```

8_17-4

```
Loading KnotTheory` version of August 22, 2010, 13:36:57.55.
Read more at http://katlas.org/wiki/KnotTheory.
```

8_17-4

KnotTheory::loading : Loading precomputed data in PD4Knots`.

8_17-4

$$-\frac{1-4\,T_1+8\,T_1^2-11\,T_1^3+8\,T_1^4-4\,T_1^5+T_1^6}{T_1^3}$$