Dancso@GRT: Categorifications of the characteristic polynomial of hyperplane arrangements

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Hyperplane Arrangements

V - k-dim v.s. over $\mathbb{k}$,

$\mathcal{V} = \{\nu_1 \dots \nu_n\} \subset V^*$ defines a hyperplane arrangement in V:

$$\mathcal{A} = \{V; H_i = \ker \nu_i\}$$

Characteristic Polynomial:

$$\chi(\mathcal{A}, t) = \sum_{S' \subseteq [n]} (-1)^{|S'|} t^{\dim H_{S'}}$$

$$H_{S'} = \bigcap_{s \in S'} H_s$$

Graphs embed in hyperplane arrangements:

if $e_i = (v_k, v_\ell)$, take $H_i = \ker(x_k - x_\ell)$

Goal. Categorify $\chi(\mathcal{A}, t)$ - find a cohomology theory s.t.

$$\chi(\mathcal{A}, t) = \chi_q(H^*(\mathcal{A}))$$

maybe some

variable
Substitution

$$\begin{tikzcd}
 & \Lambda^\bullet H_1^* \arrow[r] \arrow[dr] & \Lambda^\bullet (H_1 \cap H_2)^* \arrow[dr] & \\
\Lambda^\bullet (\mathbb{C}^3)^* \arrow[ur] \arrow[r] \arrow[dr] & \Lambda^\bullet H_2^* \arrow[ur] \arrow[dr] & \Lambda^\bullet (H_1 \cap H_3)^* \arrow[r] & \Lambda^\bullet (H_1 \cap H_2 \cap H_3)^* \\
 & \Lambda^\bullet H_3^* \arrow[r] \arrow[ur] & \Lambda^\bullet (H_2 \cap H_3)^* \arrow[ur] &
\end{tikzcd}$$

....> get a differential.