

Pensieve Header: Experimenting the “reverse double” in β -calculus.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-01"];
<< betaCalculus.m
<< KnotTheory`
GC[K_] := GC @@ (
  PD[K] /. X[i_, j_, k_, l_] => If[PositiveQ[X[i, j, k, l]],
    Ar[l, i, +1], Ar[j, i, -1]
  ]
)
```

Loading KnotTheory` version of August 22, 2010, 13:36:57.55.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
bm[x_, y_, z_][β_] := β // conj[x, y] // hm[x, y, z] // tm[x, y, z];

(β_)~BB~(ν_) := Module[
  {ρ, σ, labels},
  ρ = β + (ν /. {h[s_] => h[σ[s]], t[s_] => t[σ[s]], c[s_] => c[σ[s]]});
  labels = Union[Cases[{β, ν}, h[s_] | t[s_] | c[s_] => s, Infinity]];
  Do[
    ρ = ρ // bm[s, σ[s], s],
    {s, labels}
  ];
  ρ
]
```

$(W[1] + ar[1, 2]) \sim_{BB} (W[1] + ar[1, 3]) // \beta Form$

$$\begin{pmatrix} W[1] & h[2] & h[3] \\ t[1] & 1 & 1 \end{pmatrix}$$

$\{(W[1] + ar[1, 3]) \sim_{BB} (W[1] + ar[2, 3]),$
 $(W[1] + ar[2, 3]) \sim_{BB} (W[1] + ar[1, 3])\} // \beta Form$

$$\left\{ \begin{pmatrix} W[1] & h[3] \\ t[1] & 1 \\ t[2] & 1+c[1] \end{pmatrix}, \begin{pmatrix} W[1] & h[3] \\ t[1] & 1+c[2] \\ t[2] & 1 \end{pmatrix} \right\}$$

$(W[1] + ar[1, 2]) ** (W[1] + ar[2, 3]) // \beta Form$

$$\begin{pmatrix} W[1] & h[2] & h[3] \\ t[1] & 1 & \frac{c[2]}{1+c[1]} \\ t[2] & 0 & \frac{1}{1+c[1]} \end{pmatrix}$$

$(W[1] + ar[1, 2]) \sim_{BB} (W[1] + ar[2, 3]) // \beta Form$

$$\begin{pmatrix} W[1] & h[2] & h[3] \\ t[1] & 1 & 0 \\ t[2] & 0 & 1 \end{pmatrix}$$

$(W[1] + ar[2, 3]) \sim BB \sim (W[1] + ar[1, 2]) // \beta Form$

$$\begin{pmatrix} W[1] & h[2] & h[3] \\ t[1] & 1 & -c[2] \\ t[2] & 0 & 1+c[1] \end{pmatrix}$$

$\{ar[1, 2] \sim BB \sim ar[1, 3] \sim BB \sim ar[2, 3], ar[2, 3] \sim BB \sim ar[1, 3] \sim BB \sim ar[1, 2]\} // \beta Form$

$$\left\{ \begin{pmatrix} 0 & h[2] & h[3] \\ t[1] & 1 & 1 \\ t[2] & 0 & 1+c[1] \end{pmatrix}, \begin{pmatrix} 0 & h[2] & h[3] \\ t[1] & 1 & 1 \\ t[2] & 0 & 1+c[1] \end{pmatrix} \right\}$$

$\{\beta_3 = W[1] + \text{Sum}[\alpha_{10\ i+j}[c[1], c[2], c[3]] ar[i, j], \{i, 3\}, \{j, 3\}],$

$\beta_3 // bm[1, 2, 1]$

$\} /. e_{-}[c[1], c[1], c[3]] \Rightarrow e // \beta Form // ColumnForm$

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11}[c[1], c[2], c[3]] & \alpha_{12}[c[1], c[2], c[3]] & \alpha_{13}[c[1], c[2], c[3]] \\ t[2] & \alpha_{21}[c[1], c[2], c[3]] & \alpha_{22}[c[1], c[2], c[3]] & \alpha_{23}[c[1], c[2], c[3]] \\ t[3] & \alpha_{31}[c[1], c[2], c[3]] & \alpha_{32}[c[1], c[2], c[3]] & \alpha_{33}[c[1], c[2], c[3]] \end{pmatrix}$$

$$\begin{pmatrix} W[1+c[1] \alpha_{12}] \\ t[1] & \frac{\alpha_{11} \alpha_{12} + 2 c[1] \alpha_{11} \alpha_{12} + c[1] \alpha_{12}^2 + c[1]^2 \alpha_{11} \alpha_{12}^2 + \alpha_{21} + 2 c[1] \alpha_{12} \alpha_{21} + c[1]^2 \alpha_{12}^2 \alpha_{21} + \alpha_{22} + c[1] \alpha_{11} \alpha_{22} + c[1] \alpha_{12} \alpha_{22} + c[1]}{1} \\ t[3] \end{pmatrix}$$

$(W[1] + ar[1, 2]) \sim BB \sim \left(W[1] + \frac{-1}{1+c[1]} ar[1, 2] \right) // \beta Form$

$(W[1])$

$\{\beta_1 = W[1] + \text{Sum}[\alpha_{10\ i+j} ar[i, j], \{i, 3\}, \{j, 3\}] + ar[5, 6],$

$\beta_1 // bm[1, 5, 1],$

$\beta_1 // bm[2, 6, 2],$

$\beta_1 // bm[1, 5, 1] // dm[2, 6, 2]$

$\} /. c[i_] \Rightarrow c_i // \beta Form // ColumnForm$

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] & h[6] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} & 0 \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} & 0 \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} & 0 \\ t[5] & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] & h[6] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} & 1 \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} & 0 \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} & 0 \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t[2] & (1+c_5) \alpha_{21} & (1+c_5) \alpha_{22} & (1+c_5) \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \\ t[5] & -c_2 \alpha_{21} & 1+c_1 \alpha_{12} + c_3 \alpha_{32} & -c_2 \alpha_{23} \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & 1+\alpha_{12}+c_1 \alpha_{12}+c_2 \alpha_{22}+c_3 \alpha_{32} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

$$\{\beta_1 = W[1] + \text{Sum}[\alpha_{10\ i+j} \text{ar}[i, j], \{i, 3\}, \{j, 3\}],$$

$$\beta_1 \sim \text{BB} \sim \text{ar}[1, 2],$$

$$\beta_1 \sim \text{BB} \sim \left(\frac{-1}{1 + c[1]} \text{ar}[1, 2] \right)$$

} /. c[i_] => c_i // β Form // ColumnForm

$$\left(\begin{array}{cccc} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{array} \right)$$

$$\left(\begin{array}{cccc} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} - c_2 \alpha_{21} & 1 + \alpha_{12} + c_1 \alpha_{12} + c_3 \alpha_{32} & \alpha_{13} - c_2 \alpha_{23} \\ t[2] & (1 + c_1) \alpha_{21} & (1 + c_1) \alpha_{22} & (1 + c_1) \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{array} \right)$$

$$\left(\begin{array}{cccc} W[1] & h[1] & h[2] & h[3] \\ t[1] & \frac{\alpha_{11} + c_1 \alpha_{11} + c_2 \alpha_{21}}{1 + c_1} & \frac{-1 + \alpha_{12} - c_3 \alpha_{32}}{1 + c_1} & \frac{\alpha_{13} + c_1 \alpha_{13} + c_2 \alpha_{23}}{1 + c_1} \\ t[2] & \frac{\alpha_{21}}{1 + c_1} & \frac{\alpha_{22}}{1 + c_1} & \frac{\alpha_{23}}{1 + c_1} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{array} \right)$$

$$\{\beta_1 = W[1] + \text{Sum}[\alpha_{10\ i+j} \text{ar}[i, j], \{i, 3\}, \{j, 3\}],$$

$$\text{ar}[1, 2] \sim \text{BB} \sim \beta_1,$$

$$\left(\frac{-1}{1 + c[1]} \text{ar}[1, 2] \right) \sim \text{BB} \sim \beta_1$$

} /. c[i_] => c_i // β Form // ColumnForm

$$\left(\begin{array}{cccc} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{array} \right)$$

$$\left(\begin{array}{cccc} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & 1 + \alpha_{12} + c_1 \alpha_{12} + c_2 \alpha_{21} + c_3 \alpha_{31} & \alpha_{13} \\ t[2] & \alpha_{21} & -c_1 \alpha_{21} + \alpha_{22} + c_1 \alpha_{22} & \alpha_{23} \\ t[3] & \alpha_{31} & -c_1 \alpha_{31} + \alpha_{32} + c_1 \alpha_{32} & \alpha_{33} \end{array} \right)$$

$$\left(\begin{array}{cccc} W[1] & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \frac{-1 + \alpha_{12} - c_2 \alpha_{21} - c_3 \alpha_{31}}{1 + c_1} & \alpha_{13} \\ t[2] & \alpha_{21} & \frac{c_1 \alpha_{21} + \alpha_{22}}{1 + c_1} & \alpha_{23} \\ t[3] & \alpha_{31} & \frac{c_1 \alpha_{31} + \alpha_{32}}{1 + c_1} & \alpha_{33} \end{array} \right)$$

```

n = 2;
{
   $\alpha_1 = W[1] + \text{Sum}[\alpha_{10\ i+j} \text{ar}[i, j], \{i, n\}, \{j, n\}],$ 
   $\beta_1 = W[1] + \text{Sum}[\beta_{10\ i+j} \text{ar}[i, j], \{i, n\}, \{j, n\}],$ 
   $\alpha_1 \sim BB \sim \beta_1$ 
} /. c[i_] := c_i //  $\beta$ Form // ColumnForm

( W[1] h[1] h[2] )
( t[1]  $\alpha_{11}$   $\alpha_{12}$  )
( t[2]  $\alpha_{21}$   $\alpha_{22}$  )

( W[1] h[1] h[2] )
( t[1]  $\beta_{11}$   $\beta_{12}$  )
( t[2]  $\beta_{21}$   $\beta_{22}$  )

( W[1] h[1] h[2] )
( t[1]  $\alpha_{11} + \beta_{11} + c_1 \alpha_{11} \beta_{11} + c_2 \alpha_{21} \beta_{11} - c_2 \alpha_{21} \beta_{12} + c_2 \alpha_{11} \beta_{21}$   $\alpha_{12} + \beta_{12} + c_1 \alpha_{12} \beta_{12} + c_2 \alpha_1$  )
( t[2]  $\alpha_{21} + c_1 \alpha_{21} \beta_{12} + \beta_{21} + c_2 \alpha_{21} \beta_{21}$   $\alpha_{22} + c_1 \alpha_{22} \beta_{12} - c_1 \alpha_{12} \beta_{21} + \beta_{22} + c_1 \alpha_1$  )

```

```

n = 3;
{
   $\alpha_1 = W[1] + \text{Sum}[\alpha_{10\ i+j} \text{ar}[i, j], \{i, n\}, \{j, n\}],$ 
   $\beta_1 = W[1] + \text{Sum}[\beta_{10\ i+j} \text{ar}[i, j], \{i, n\}, \{j, n\}],$ 
   $\alpha_1 \sim BB \sim \beta_1$ 
} /. c[i_] := c_i //  $\beta$ Form // ColumnForm

( W[1] h[1] h[2] h[3] )
( t[1]  $\alpha_{11}$   $\alpha_{12}$   $\alpha_{13}$  )
( t[2]  $\alpha_{21}$   $\alpha_{22}$   $\alpha_{23}$  )
( t[3]  $\alpha_{31}$   $\alpha_{32}$   $\alpha_{33}$  )

( W[1] h[1] h[2] h[3] )
( t[1]  $\beta_{11}$   $\beta_{12}$   $\beta_{13}$  )
( t[2]  $\beta_{21}$   $\beta_{22}$   $\beta_{23}$  )
( t[3]  $\beta_{31}$   $\beta_{32}$   $\beta_{33}$  )

( W[1] h[1] )
( t[1]  $\alpha_{11} + \beta_{11} + c_1 \alpha_{11} \beta_{11} + c_2 \alpha_{21} \beta_{11} + c_3 \alpha_{31} \beta_{11} - c_2 \alpha_{21} \beta_{12} - c_3 \alpha_{31} \beta_{13} + c_2 \alpha_{11} \beta_{21} + c_3 \alpha_{11} \beta_{31}$  )
( t[2]  $\alpha_{21} + c_1 \alpha_{21} \beta_{12} + \beta_{21} + c_2 \alpha_{21} \beta_{21} + c_3 \alpha_{31} \beta_{21} - c_3 \alpha_{31} \beta_{23} + c_3 \alpha_{21} \beta_{32}$  )
( t[3]  $\alpha_{31} + c_1 \alpha_{31} \beta_{13} + c_2 \alpha_{31} \beta_{23} + \beta_{31} + c_2 \alpha_{21} \beta_{31} + c_3 \alpha_{31} \beta_{31} - c_2 \alpha_{21} \beta_{32}$  )

```

```

{K = Knot[8, 17];
 Alexander[K][X], GC[K]
}

```

KnotTheory:loading: Loading precomputed data in PD4Knots`.

$$\left\{ 11 - \frac{1}{X^3} + \frac{4}{X^2} - \frac{8}{X} - 8X + 4X^2 - X^3, \text{GC}[\text{Ar}[1, 6, 1], \text{Ar}[7, 14, 1], \text{Ar}[3, 8, -1], \text{Ar}[13, 2, -1], \text{Ar}[5, 12, -1], \text{Ar}[9, 4, -1], \text{Ar}[11, 16, 1], \text{Ar}[15, 10, 1]] \right\}$$

```

βAlex[K_] := Module[
  {gc, β},
  gc = GC[K];
  β = βCollect[
    Plus@@(gc /. {Ar[i_, j_, +1] => R[i, j], Ar[i_, j_, -1] => RInv[i, j]})];
  Do[β = bm[1, k, 1][β], {k, 2, 2 Length[gc]}];
  Expand[β /. h[1] → 0 /. E^(n_. c[1]) => X^n /. W[a_] => a]
]

```

```
βAlex[K]
```

$$-8 - \frac{1}{X^2} + \frac{4}{X} + 11X - 8X^2 + 4X^3 - X^4$$

```
βSimplify = Factor;
```

```
gc = GC[K];
```

```

Join[{{β = βCollect[
  W[1] + Plus@@
  (gc /. {Ar[i_, j_, +1] => ar[i, j], Ar[i_, j_, -1] => -\frac{1}{1+c[i]} ar[i, j]})
]},
  Table[β = bm[1, k, 1][β], {k, 2, 2 Length[gc]}]
] /. _c → c - 1 // βCollect // βForm // ColumnForm

```

W[1]	h[2]	h[4]	h[6]	h[8]	h[10]	h[12]	h[14]	h[16]
t[1]	0	0	1	0	0	0	0	0
t[3]	0	0	0	$-\frac{1}{c}$	0	0	0	0
t[5]	0	0	0	0	0	$-\frac{1}{c}$	0	0
t[7]	0	0	0	0	0	0	1	0
t[9]	0	$-\frac{1}{c}$	0	0	0	0	0	0
t[11]	0	0	0	0	0	0	0	1
t[13]	$-\frac{1}{c}$	0	0	0	0	0	0	0
t[15]	0	0	0	0	1	0	0	0

W[1]	h[1]	h[4]	h[6]	h[8]	h[10]	h[12]	h[14]	h[16]
t[1]	0	0	$\frac{1}{c}$	0	0	0	0	0
t[3]	0	0	0	$-\frac{1}{c}$	0	0	0	0
t[5]	0	0	0	0	0	$-\frac{1}{c}$	0	0
t[7]	0	0	0	0	0	0	1	0
t[9]	0	$-\frac{1}{c}$	0	0	0	0	0	0
t[11]	0	0	0	0	0	0	0	1
t[13]	$-\frac{1}{c}$	0	$\frac{-1+c}{c}$	0	0	0	0	0
t[15]	0	0	0	0	1	0	0	0

$$\begin{pmatrix} W[1] & h[1] & h[4] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & 0 & \frac{1}{c} & -\frac{1}{c} & 0 & 0 & 0 & 0 \\ t[5] & 0 & 0 & 0 & 0 & 0 & -\frac{1}{c} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ t[9] & 0 & -\frac{1}{c} & 0 & 0 & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ t[13] & -\frac{1}{c} & 0 & \frac{-1+c}{c} & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & \frac{1}{c^2} & -\frac{1}{c^2} & 0 & 0 & 0 & 0 \\ t[5] & 0 & 0 & 0 & 0 & -\frac{1}{c} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ t[9] & -\frac{1}{c^2} & \frac{-1+c}{c^2} & -\frac{-1+c}{c^2} & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ t[13] & -\frac{1}{c} & \frac{-1+c}{c} & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & \frac{1}{c^2} & -\frac{1}{c^2} & 0 & -\frac{1}{c} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ t[9] & -\frac{1}{c^2} & \frac{-1+c}{c^2} & -\frac{-1+c}{c^2} & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ t[13] & -\frac{1}{c} & \frac{-1+c}{c} & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} W\left[\frac{-1+c+c^2}{c^2}\right] & h[1] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & \frac{1}{c(-1+c+c^2)} & -\frac{c}{-1+c+c^2} & 0 & -\frac{c^2}{-1+c+c^2} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 1 & 0 \\ t[9] & -\frac{1}{-1+c+c^2} & -\frac{-1+c}{-1+c+c^2} & 0 & \frac{(-1+c)^2}{c(-1+c+c^2)} & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 1 \\ t[13] & -\frac{c}{-1+c+c^2} & \frac{(-1+c)^2}{c(-1+c+c^2)} & 0 & \frac{(-1+c)^2}{-1+c+c^2} & 0 & 0 \\ t[15] & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} W\left[\frac{-1+c+c^2}{c^2}\right] & h[1] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & \frac{1}{c(-1+c+c^2)} & -\frac{c}{-1+c+c^2} & 0 & -\frac{c^2}{-1+c+c^2} & 1 & 0 \\ t[9] & -\frac{1}{-1+c+c^2} & -\frac{-1+c}{-1+c+c^2} & 0 & \frac{(-1+c)^2}{c(-1+c+c^2)} & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 1 \\ t[13] & -\frac{c}{-1+c+c^2} & \frac{(-1+c)^2}{c(-1+c+c^2)} & 0 & \frac{(-1+c)^2}{-1+c+c^2} & 0 & 0 \\ t[15] & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[\frac{-1+2c}{c^2}\right] & h[1] & h[10] & h[12] & h[14] & h[16] \\
 t[1] & -\frac{-1+c}{c^2(-1+2c)} & 0 & -\frac{c}{-1+2c} & \frac{-1+c+c^2}{c(-1+2c)} & 0 \\
 t[9] & -\frac{1}{-1+2c} & 0 & -\frac{(-1+c)^3}{c(-1+2c)} & \frac{(-1+c)^2}{-1+2c} & 0 \\
 t[11] & 0 & 0 & 0 & 0 & 1 \\
 t[13] & -\frac{1}{-1+2c} & 0 & \frac{(-1+c)^2}{-1+2c} & -\frac{(-1+c)^3}{c(-1+2c)} & 0 \\
 t[15] & 0 & 1 & 0 & 0 & 0
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[\frac{-1+2c}{c^2}\right] & h[1] & h[10] & h[12] & h[14] & h[16] \\
 t[1] & -\frac{-1+c+c^2}{c^2(-1+2c)} & 0 & -\frac{-1+3c-2c^2+c^3}{c(-1+2c)} & \frac{-1+2c-c^2+c^3}{c(-1+2c)} & 0 \\
 t[11] & 0 & 0 & 0 & 0 & 1 \\
 t[13] & -\frac{1}{-1+2c} & 0 & \frac{(-1+c)^2}{-1+2c} & -\frac{(-1+c)^3}{c(-1+2c)} & 0 \\
 t[15] & 0 & 1 & 0 & 0 & 0
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[\frac{-1+2c}{c^2}\right] & h[1] & h[12] & h[14] & h[16] \\
 t[1] & -\frac{-1+c+c^2}{c(-1+2c)} & -\frac{-1+3c-2c^2+c^3}{-1+2c} & \frac{-1+2c-c^2+c^3}{-1+2c} & 0 \\
 t[11] & 0 & 0 & 0 & 1 \\
 t[13] & -\frac{1}{-1+2c} & \frac{(-1+c)^2}{-1+2c} & -\frac{(-1+c)^3}{c(-1+2c)} & 0 \\
 t[15] & \frac{c}{-1+2c} & \frac{(-1+c)(-1+3c-2c^2+c^3)}{c(-1+2c)} & -\frac{(-1+c)(-1+2c-c^2+c^3)}{c(-1+2c)} & 0
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[\frac{-1+2c}{c^2}\right] & h[1] & h[12] & h[14] & h[16] \\
 t[1] & -\frac{-1+c+c^2}{c(-1+2c)} & -\frac{-1+3c-2c^2+c^3}{-1+2c} & \frac{-1+2c-c^2+c^3}{-1+2c} & 1 \\
 t[13] & -\frac{1}{-1+2c} & \frac{(-1+c)^2}{-1+2c} & -\frac{(-1+c)^3}{c(-1+2c)} & 0 \\
 t[15] & \frac{c}{-1+2c} & \frac{(-1+c)(-1+3c-2c^2+c^3)}{c(-1+2c)} & -\frac{(-1+c)(-1+2c-c^2+c^3)}{c(-1+2c)} & 0
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[-\frac{2-6c+5c^2-3c^3+c^4}{c^2}\right] & h[1] & h[14] & h[16] \\
 t[1] & \frac{-2+4c-c^2+c^3}{c^2(2-6c+5c^2-3c^3+c^4)} & -\frac{-1+2c-c^2+c^3}{c(2-6c+5c^2-3c^3+c^4)} & -\frac{-1+2c}{c(2-6c+5c^2-3c^3+c^4)} \\
 t[13] & -\frac{-2+2c-2c^2+c^3}{2-6c+5c^2-3c^3+c^4} & \frac{(-1+c)^3(2-c+c^2)}{c(2-6c+5c^2-3c^3+c^4)} & \frac{(-1+c)^3}{2-6c+5c^2-3c^3+c^4} \\
 t[15] & -\frac{c}{2-6c+5c^2-3c^3+c^4} & \frac{(-1+c)(-1+2c-c^2+c^3)}{c(2-6c+5c^2-3c^3+c^4)} & \frac{(-1+c)^2(-1+3c-2c^2+c^3)}{c(2-6c+5c^2-3c^3+c^4)}
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[-\frac{2-6c+5c^2-3c^3+c^4}{c^2}\right] & h[1] & h[14] & h[16] \\
 t[1] & -\frac{2-4c-c^2+c^3-2c^4+c^5}{c^2(2-6c+5c^2-3c^3+c^4)} & \frac{-1+5c-9c^2+7c^3-4c^4+c^5}{c(2-6c+5c^2-3c^3+c^4)} & \frac{1-3c+3c^2-3c^3+c^4}{c(2-6c+5c^2-3c^3+c^4)} \\
 t[15] & -\frac{c}{2-6c+5c^2-3c^3+c^4} & \frac{(-1+c)(-1+2c-c^2+c^3)}{c(2-6c+5c^2-3c^3+c^4)} & \frac{(-1+c)^2(-1+3c-2c^2+c^3)}{c(2-6c+5c^2-3c^3+c^4)}
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[-\frac{1-4c+8c^2-11c^3+8c^4-4c^5+c^6}{c^3}\right] & h[1] & h[16] \\
 t[1] & -\frac{1-3c+5c^2-8c^3+5c^4-3c^5+c^6}{c(1-4c+8c^2-11c^3+8c^4-4c^5+c^6)} & \frac{c(1-3c+3c^2-3c^3+c^4)}{1-4c+8c^2-11c^3+8c^4-4c^5+c^6} \\
 t[15] & \frac{1-3c+3c^2-3c^3+c^4}{1-4c+8c^2-11c^3+8c^4-4c^5+c^6} & \frac{(-1+c)^4(1-c+c^2)}{1-4c+8c^2-11c^3+8c^4-4c^5+c^6}
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[-\frac{1-4c+8c^2-11c^3+8c^4-4c^5+c^6}{c^3}\right] & h[1] & h[16] \\
 t[1] & -\frac{1}{c} & 1
 \end{pmatrix}$$

$$\begin{pmatrix}
 W\left[-\frac{1-4c+8c^2-11c^3+8c^4-4c^5+c^6}{c^2}\right]
 \end{pmatrix}$$