Pensieve Header: The pentagon in β-calculus, ignoring the hexagon.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-01"];
<< betaCalculus.m
```

```
(V = α ar[1, 1] + β ar[1, 2] + γ ar[2, 1] + δ ar[2, 2]) // βForm
```

$$\begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & \delta \end{pmatrix}$$

```
({t1, t2} = {V ** dΔ[1, 1, 2][R[1, 3]], R[1, 3] ** R[2, 3] ** V} /. _W → 0) // βForm
```

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & \alpha & \beta & \frac{\left(-1+e^{c[1]+c[2]}\right)\left(1+\alpha c[1]+\beta c[1]+\alpha\beta c[1]^2+\beta c[2]+\delta c[2]+\alpha\beta c[1]c[2]+\alpha\delta c[1]c[2]+\beta\gamma c[2]^2\right)}{(c[1]+c[2])(1+\alpha c[1]+\gamma c[2])(1+\beta c[1]+\delta c[2])} \\ t[2] & \gamma & \delta & \frac{\left(-1+e^{c[1]+c[2]}\right)\left(1+\alpha c[1]+\gamma c[1]+\beta\gamma c[1]^2+\gamma c[2]+\delta c[2]+\alpha\delta c[1]c[2]+\gamma\delta c[1]c[2]+\gamma\delta c[2]^2\right)}{(c[1]+c[2])(1+\alpha c[1]+\gamma c[2])(1+\beta c[1]+\delta c[2])} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & \alpha & \beta & \frac{-1+e^{c[1]}}{c[1]} \\ t[2] & \gamma & \delta & \frac{e^{c[1]}\left(-1+e^{c[2]}\right)}{c[2]} \end{pmatrix}\right\}$$

```
eqns = Simplify[Coefficient[(t1 - t2 // βCollect) /. h[3] → 1, t[#]] == 0] & /@ {1, 2}
```

$$\left\{\left(-e^{c[1]}\,(c[1]+c[2])\,(1+\alpha c[1]+\gamma c[2])\,(1+\beta c[1]+\delta c[2])+c[2]\left(1+\delta c[2]+\alpha c[1]\,(1+\delta c[2])+\gamma\left(c[1]+\beta c[1]^2+c[2]+\delta c[1]c[2]+\delta c[2]^2\right)\right)+e^{c[1]+c[2]}\,c[1]\left(1+\delta c[2]+\beta\left(c[1]+c[2]+\gamma c[2]^2\right)+\alpha c[1]\,(1+\delta c[2]+\beta\,(c[1]+c[2]))\right)\right)\Big/\left(c[1]\,(c[1]+c[2])\,(1+\alpha c[1]+\gamma c[2])\,(1+\beta c[1]+\delta c[2])\right)==0,$$

$$\left(e^{c[1]}\,(c[1]+c[2])\,(1+\alpha c[1]+\gamma c[2])\,(1+\beta c[1]+\delta c[2])-c[2]\left(1+\delta c[2]+\alpha c[1]\,(1+\delta c[2])+\gamma\left(c[1]+\beta c[1]^2+c[2]+\delta c[1]c[2]+\delta c[2]^2\right)\right)-e^{c[1]+c[2]}\,c[1]\left(1+\delta c[2]+\beta\left(c[1]+c[2]+\gamma c[2]^2\right)+\alpha c[1]\,(1+\delta c[2]+\beta\,(c[1]+c[2]))\right)\right)\Big/\left(c[2]\,(c[1]+c[2])\,(1+\alpha c[1]+\gamma c[2])\,(1+\beta c[1]+\delta c[2])\right)==0\Big\}$$

```
γsol = (Solve[eqns, {α, β, γ, δ}] /. {α | β | δ → 0})[[1, 1, 2]] // FullSimplify
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\frac{e^{c[1]}\left(\left(-1+e^{c[2]}\right)c[1]-c[2]\right)+c[2]}{\left(-1+e^{c[1]}\right)c[2]\,(c[1]+c[2])}$$

```
(V0 = βCollect[V /. {γ → γsol, α | β | δ → 0}]) // βForm
```

$$\begin{pmatrix} 0 & h[1] \\ t[2] & -\frac{e^{c[1]}\,c[1]-e^{c[1]+c[2]}\,c[1]-c[2]+e^{c[1]}\,c[2]}{\left(-1+e^{c[1]}\right)c[2]\,(c[1]+c[2])} \end{pmatrix}$$

```
V0 ** dΔ[1, 1, 2][R[1, 3]] - R[1, 3] ** R[2, 3] ** V0 // βForm
```

$$(W[1])$$

```
(V0Inv = λ ar[2, 1]) // βForm
```

$$\begin{pmatrix} 0 & h[1] \\ t[2] & \lambda \end{pmatrix}$$

```
(V0Inv ** V0) // βForm
```

$$\begin{pmatrix} 0 & h[1] \\ t[2] & \frac{-e^{c[1]}\,c[1]+e^{c[1]+c[2]}\,c[1]+c[2]-e^{c[1]}\,c[2]-\lambda\,c[1]\,c[2]+e^{c[1]+c[2]}\,\lambda\,c[1]\,c[2]}{\left(-1+e^{c[1]}\right)\,c[2]\,(c[1]+c[2])} \end{pmatrix}$$

```
λsol = First@Solve[λ + (E^c[1] ((-1 + E^c[2]) c[1] - c[2]) + c[2]) (1 + λ c[2]) / ((-1 + E^c[1]) c[2] (c[1] + c[2])) == 0, λ]
```

$$\left\{\lambda \to \frac{e^{c[1]}\,c[1]-e^{c[1]+c[2]}\,c[1]-c[2]+e^{c[1]}\,c[2]}{\left(-1+e^{c[1]+c[2]}\right)\,c[1]\,c[2]}\right\}$$

```
(V0Inv = V0Inv /. λsol) // βForm
```

$$\begin{pmatrix} 0 & h[1] \\ t[2] & -\frac{-e^{c[1]}\,c[1]+e^{c[1]+c[2]}\,c[1]+c[2]-e^{c[1]}\,c[2]}{\left(-1+e^{c[1]+c[2]}\right)\,c[1]\,c[2]} \end{pmatrix}$$

```
(Φ =
   (V0Inv // dP[2 → 3] // dΔ[1, 1, 2]) **
    V0Inv ** (V0 // dP[1 → 2, 2 → 3]) ** (V0 // dΔ[2, 2, 3])
 ) //
 βForm
```

$$\begin{pmatrix} 0 & \\ t[2] & -\frac{-e^{c[2]}\,c[1]\,c[2]+e^{c[1]+c[2]}\,c[1]\,c[2]+e^{c[2]+c[3]}\,c[1]\,c[2]-e^{c[1]+c[2]+c[3]}\,c[1]\,c[2]-e^{c[2]}\,c[2]^2+e^{c[1]+c[2]}\,c[2]^2+e^{c[2]+c[3]}\,c[2]^2-e^{c}}{\left(-1+e\right.} \\ t[3] & \frac{-e^{c[2]}\,c[1]\,c[2]+e^{c[1]+c[2]}\,c[1]\,c[2]+e^{c[2]+c[3]}\,c[1]\,c[2]-e^{c[1]+c[2]+c[3]}\,c[1]\,c[2]-e^{c[2]}\,c[2]^2+e^{c[1]+c[2]}\,c[2]^2+e^{c[2]+c[3]}\,c[2]^2-e^{c[}}{\left(-1+e^{c}\right.} \end{pmatrix}$$

```
(t1 = Φ ** (Φ // dP[3 → 4] // dΔ[2, 2, 3]) ** (Φ // dP[1 → 2, 2 → 3, 3 → 4])) // βForm
```

$$\begin{pmatrix} 0 & \\ t[2] & \\ t[3] & \frac{e^{c[2]}\,c[1]\,c[2]-e^{c[1]+c[2]}\,c[1]\,c[2]-e^{c[2]+c[3]}\,c[1]\,c[2]+e^{c[1]+c[2]+c[3]}\,c[1]\,c[2]-e^{c[1]+2\,c[2]+c[3]+c[4]}\,c[1]\,c[2]+e^{2\,c[1]+2\,c[2]+c[3]+c[4]}} \\ t[4] & \end{pmatrix}$$

```
(t2 = (Φ // dP[2 → 3, 3 → 4] // dΔ[1, 1, 2]) ** (Φ // dΔ[3, 3, 4])) // βForm
```

$$\begin{pmatrix} 0 & \\ t[2] & \\ t[3] & \frac{e^{c[2]}\,c[1]\,c[2]-e^{c[1]+c[2]}\,c[1]\,c[2]-e^{c[2]+c[3]}\,c[1]\,c[2]+e^{c[1]+c[2]+c[3]}\,c[1]\,c[2]-e^{c[1]+2\,c[2]+c[3]+c[4]}\,c[1]\,c[2]+e^{2\,c[1]+2\,c[2]+c[3]+c[4]}} \\ t[4] & \end{pmatrix}$$

```
t1 == t2
```

```
True
```