

Pensieve header: Programming the Kauffman bracket in the conventions of <http://www.math.toronto.edu/~drorbn/papers/Categorification/QRG.pdf>.

```
<< KnotTheory`
```

```
Loading KnotTheory` version of August 22, 2010, 13:36:57.55.  
Read more at http://katlas.org/wiki/KnotTheory.
```

```
SetAttributes[p, Orderless]
```

```
q[1, 2] === q[2, 1]
```

```
False
```

```
p[1, 2] === p[2, 1]
```

```
True
```

```
pd = PD[Knot[8, 17]]
```

```
PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14],  
X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]
```

```
t1 = pd /. X[i_, j_, k_, l_] => p[i, j] p[k, l] - q p[i, l] p[j, k]
```

```
PD[p[1, 7] p[2, 6] - q p[1, 6] p[2, 7], p[7, 15] p[8, 14] - q p[7, 14] p[8, 15],  
-q p[3, 9] p[4, 8] + p[3, 8] p[4, 9], -q p[2, 14] p[3, 13] + p[2, 13] p[3, 14],  
-q p[5, 13] p[6, 12] + p[5, 12] p[6, 13], -q p[4, 10] p[5, 9] + p[4, 9] p[5, 10],  
-q p[1, 12] p[11, 16] + p[1, 11] p[12, 16], p[10, 16] p[11, 15] - q p[10, 15] p[11, 16]]
```

```
t2 = Expand[t1 /. PD -> Times]
```

A very large output was generated. Here is a sample of it:

```
-q5 p[1, 7] p[1, 12] p[2, 6] p[2, 14] p[3, 9] p[3, 13] p[4, 8] p[4, 10] p[5, 9]  
p[5, 13] p[6, 12] p[7, 15] p[8, 14] p[10, 16] p[11, 15] p[11, 16] + <<382>>
```

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```
t3 = t2 //. p[i_, j_] p[j_, k_] => p[i, k]
```

$$\begin{aligned}
& q^8 p[6, 12]^2 p[7, 14]^2 p[9, 13]^2 p[10, 15]^2 p[11, 16]^2 - \\
& q^7 p[7, 14]^2 p[10, 15]^2 p[11, 16]^2 p[12, 13]^2 - q^5 p[6, 13]^2 p[10, 15]^2 p[11, 16]^2 p[12, 14]^2 - \\
& q^7 p[9, 13]^2 p[10, 15]^2 p[11, 16]^2 p[12, 14]^2 - q^3 p[6, 13]^2 p[8, 14]^2 p[11, 16]^2 p[12, 15]^2 + \\
& q^2 p[4, 9]^2 p[6, 13]^2 p[8, 14]^2 p[11, 16]^2 p[12, 15]^2 - \\
& q^3 p[6, 13]^2 p[9, 14]^2 p[11, 16]^2 p[12, 15]^2 - \\
& q^7 p[7, 14]^2 p[9, 13]^2 p[10, 15]^2 p[12, 16]^2 - qp[6, 13]^2 p[8, 14]^2 p[11, 15]^2 p[12, 16]^2 + \\
& p[4, 9]^2 p[6, 13]^2 p[8, 14]^2 p[11, 15]^2 p[12, 16]^2 - \\
& qp[6, 13]^2 p[9, 14]^2 p[11, 15]^2 p[12, 16]^2 + 3q^6 p[10, 15]^2 p[11, 16]^2 p[13, 14]^2 - \\
& q^7 p[6, 12]^2 p[10, 15]^2 p[11, 16]^2 p[13, 14]^2 + 2q^4 p[11, 16]^2 p[12, 15]^2 p[13, 14]^2 - \\
& q^3 p[4, 9]^2 p[11, 16]^2 p[12, 15]^2 p[13, 14]^2 + q^6 p[10, 15]^2 p[12, 16]^2 p[13, 14]^2 + \\
& 2q^2 p[11, 15]^2 p[12, 16]^2 p[13, 14]^2 - qp[4, 9]^2 p[11, 15]^2 p[12, 16]^2 p[13, 14]^2 + \\
& 2q^6 p[7, 14]^2 p[11, 16]^2 p[13, 15]^2 - q^5 p[4, 9]^2 p[7, 14]^2 p[11, 16]^2 p[13, 15]^2 - \\
& 2q^7 p[6, 12]^2 p[7, 14]^2 p[11, 16]^2 p[13, 15]^2 + \\
& q^6 p[4, 9]^2 p[6, 12]^2 p[7, 14]^2 p[11, 16]^2 p[13, 15]^2 + 2q^4 p[8, 14]^2 p[11, 16]^2 p[13, 15]^2 - \\
& 2q^3 p[4, 9]^2 p[8, 14]^2 p[11, 16]^2 p[13, 15]^2 - q^5 p[6, 12]^2 p[8, 14]^2 p[11, 16]^2 p[13, 15]^2 + \\
& q^4 p[4, 9]^2 p[6, 12]^2 p[8, 14]^2 p[11, 16]^2 p[13, 15]^2 + 2q^4 p[9, 14]^2 p[11, 16]^2 p[13, 15]^2 - \\
& q^5 p[6, 12]^2 p[9, 14]^2 p[11, 16]^2 p[13, 15]^2 + 2q^6 p[11, 16]^2 p[12, 14]^2 p[13, 15]^2 - \\
& q^5 p[4, 9]^2 p[11, 16]^2 p[12, 14]^2 p[13, 15]^2 + 2q^4 p[7, 14]^2 p[12, 16]^2 p[13, 15]^2 + \\
& 2q^6 p[7, 14]^2 p[12, 16]^2 p[13, 15]^2 - q^3 p[4, 9]^2 p[7, 14]^2 p[12, 16]^2 p[13, 15]^2 - \\
& q^5 p[4, 9]^2 p[7, 14]^2 p[12, 16]^2 p[13, 15]^2 + q^2 p[8, 14]^2 p[12, 16]^2 p[13, 15]^2 + \\
& q^4 p[8, 14]^2 p[12, 16]^2 p[13, 15]^2 - qp[4, 9]^2 p[8, 14]^2 p[12, 16]^2 p[13, 15]^2 - \\
& q^3 p[4, 9]^2 p[8, 14]^2 p[12, 16]^2 p[13, 15]^2 + q^2 p[9, 14]^2 p[12, 16]^2 p[13, 15]^2 + \\
& q^4 p[9, 14]^2 p[12, 16]^2 p[13, 15]^2 + q^6 p[7, 14]^2 p[10, 15]^2 p[13, 16]^2 + \\
& q^2 p[8, 14]^2 p[11, 15]^2 p[13, 16]^2 - qp[4, 9]^2 p[8, 14]^2 p[11, 15]^2 p[13, 16]^2 + \\
& q^2 p[9, 14]^2 p[11, 15]^2 p[13, 16]^2 - 13q^5 p[11, 16]^2 p[14, 15]^2 + \\
& 5q^4 p[4, 9]^2 p[11, 16]^2 p[14, 15]^2 + 5q^6 p[6, 12]^2 p[11, 16]^2 p[14, 15]^2 - \\
& 2q^5 p[4, 9]^2 p[6, 12]^2 p[11, 16]^2 p[14, 15]^2 + 3q^4 p[6, 13]^2 p[11, 16]^2 p[14, 15]^2 - \\
& q^3 p[4, 9]^2 p[6, 13]^2 p[11, 16]^2 p[14, 15]^2 + q^6 p[9, 13]^2 p[11, 16]^2 p[14, 15]^2 - \\
& q^7 p[6, 12]^2 p[9, 13]^2 p[11, 16]^2 p[14, 15]^2 + q^6 p[11, 16]^2 p[12, 13]^2 p[14, 15]^2 - \\
& 6q^3 p[12, 16]^2 p[14, 15]^2 - 5q^5 p[12, 16]^2 p[14, 15]^2 + 3q^2 p[4, 9]^2 p[12, 16]^2 p[14, 15]^2 + \\
& 2q^4 p[4, 9]^2 p[12, 16]^2 p[14, 15]^2 + 2q^2 p[6, 13]^2 p[12, 16]^2 p[14, 15]^2 - \\
& qp[4, 9]^2 p[6, 13]^2 p[12, 16]^2 p[14, 15]^2 + q^6 p[9, 13]^2 p[12, 16]^2 p[14, 15]^2 - \\
& 2q^3 p[13, 16]^2 p[14, 15]^2 - q^5 p[13, 16]^2 p[14, 15]^2 + q^2 p[4, 9]^2 p[13, 16]^2 p[14, 15]^2 - \\
& 3q^5 p[10, 15]^2 p[14, 16]^2 + q^4 p[6, 13]^2 p[10, 15]^2 p[14, 16]^2 + \\
& q^6 p[9, 13]^2 p[10, 15]^2 p[14, 16]^2 - 4q^3 p[11, 15]^2 p[14, 16]^2 + \\
& q^2 p[4, 9]^2 p[11, 15]^2 p[14, 16]^2 + q^2 p[6, 13]^2 p[11, 15]^2 p[14, 16]^2 + \\
& q^4 p[9, 13]^2 p[11, 15]^2 p[14, 16]^2 - q^3 p[13, 15]^2 p[14, 16]^2 - \\
& 2q^5 p[13, 15]^2 p[14, 16]^2 + q^4 p[4, 9]^2 p[13, 15]^2 p[14, 16]^2 + 37q^4 p[15, 16]^2 - \\
& 13q^3 p[4, 9]^2 p[15, 16]^2 - 5q^5 p[6, 12]^2 p[15, 16]^2 + 2q^4 p[4, 9]^2 p[6, 12]^2 p[15, 16]^2 - \\
& 7q^3 p[6, 13]^2 p[15, 16]^2 + 2q^2 p[4, 9]^2 p[6, 13]^2 p[15, 16]^2 - 7q^5 p[7, 14]^2 p[15, 16]^2 + \\
& 3q^4 p[4, 9]^2 p[7, 14]^2 p[15, 16]^2 + 2q^6 p[6, 12]^2 p[7, 14]^2 p[15, 16]^2 - \\
& q^5 p[4, 9]^2 p[6, 12]^2 p[7, 14]^2 p[15, 16]^2 - 5q^3 p[8, 14]^2 p[15, 16]^2 + \\
& 5q^2 p[4, 9]^2 p[8, 14]^2 p[15, 16]^2 + q^4 p[6, 12]^2 p[8, 14]^2 p[15, 16]^2 - \\
& q^3 p[4, 9]^2 p[6, 12]^2 p[8, 14]^2 p[15, 16]^2 + 2q^2 p[6, 13]^2 p[8, 14]^2 p[15, 16]^2 - \\
& 2qp[4, 9]^2 p[6, 13]^2 p[8, 14]^2 p[15, 16]^2 - 4q^5 p[9, 13]^2 p[15, 16]^2 + \\
& q^6 p[6, 12]^2 p[9, 13]^2 p[15, 16]^2 + q^6 p[7, 14]^2 p[9, 13]^2 p[15, 16]^2 - \\
& q^7 p[6, 12]^2 p[7, 14]^2 p[9, 13]^2 p[15, 16]^2 - 5q^3 p[9, 14]^2 p[15, 16]^2 + \\
& q^4 p[6, 12]^2 p[9, 14]^2 p[15, 16]^2 + 2q^2 p[6, 13]^2 p[9, 14]^2 p[15, 16]^2 - \\
& q^5 p[12, 13]^2 p[15, 16]^2 + q^6 p[7, 14]^2 p[12, 13]^2 p[15, 16]^2 - 2q^5 p[12, 14]^2 p[15, 16]^2 + \\
& q^4 p[4, 9]^2 p[12, 14]^2 p[15, 16]^2 + q^4 p[6, 13]^2 p[12, 14]^2 p[15, 16]^2 + \\
& q^6 p[9, 13]^2 p[12, 14]^2 p[15, 16]^2 - 4q^3 p[13, 14]^2 p[15, 16]^2 - 4q^5 p[13, 14]^2 p[15, 16]^2 + \\
& 2q^2 p[4, 9]^2 p[13, 14]^2 p[15, 16]^2 + q^6 p[6, 12]^2 p[13, 14]^2 p[15, 16]^2
\end{aligned}$$

```
t4 = Expand[t3 /. {p[i_, i_] -> (q+1/q), p[i_, j_]^2 -> (q+1/q)}]
```

$$\frac{1}{q^5} - \frac{2}{q^3} + \frac{2}{q} - q + q^3 + q^5 - q^7 + 2q^9 - 2q^{11} + q^{13}$$