Type Theory

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G grothendieck universe

$j(G)$ is also grothendieck universe (not really, but close enough)

$G \in j(G)$

$j(G)$ is like G $\quad a \in G \Rightarrow j(a) = a$

obvious property: φ is true in G iff φ is true in $j(G)$

$f : G \to G$ $\qquad A \subseteq G$

$j(f) : j(G) \to j(G)$ $\qquad j(A) \subseteq j(G)$

For all$_{\text{set}}$ X, there is $j(X)$

For example there is $j(j(a))$

$\forall a, b, c, \ldots, d$ $\quad \varphi(a, b, c, \ldots, d)$ iff

($\not\forall$) $\quad M \models \varphi(j(a), j(b), \ldots, j(d))$

M is a world where G isn't measurable

$A \subseteq G \Rightarrow j(A) \subseteq j(G)$ $\quad$ is $G \in j(A)$

$U = \{ A \subseteq G \mid G \in j(A) \}$

Say, you want to prove: G is so big, it has Q in it. example, another grothendeick universe. Then you look in M, and a Q out of G inside $j(G)$, then, by (*), there is a Q in G.

What is a natural number?

With an object $a : X$, and function $f : X \to X$, it is always possible to define $f^n(x)$ for a natural number n.

Converse: if it is possible to define $n(f, x) \in X$ for $x \in X$ and $f : X \to X$ and for any set X then n corresponds to a natural number

Def.

A natural number is something which turns a set X and a function $f : X \to X$ into another function $f^n : X \to X$

This is what they do in system F

System F:

Types: There type variables $X, Y, Z, \ldots$

if A & B are types, $A \to B$ is a type

If A is a type and X is type variable, $\Pi X. A$ is a type

eg. $\Pi X. (X \to X) \to (X \to X)$ is the type of all natural numbers, as above