Licata: Quantum affine algebras, braid groups, and Heisenberg categorification

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2:08 PM

"Higher Rep. Theory (categorification) for affine Lie algebras"

Joint w/ Cautis & Josh Sussan.

Let $\mathfrak{g}$ be a f.d. simple Lie alg. of type ADE

$\hat{\mathfrak{g}} = \mathfrak{g} \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c$ "the affine Lie algebra"

Let V_{Λ_0} be the "basic rep" — characterised by

(1) irreducible.

(2) $\exists\, v \in V_{\Lambda_0}$ that generates all: $U(\hat{\mathfrak{g}})v = V_{\Lambda_0}$

yet

(3) $\mathfrak{g} \otimes \mathbb{C}[t] \cdot v = 0$

(4) $c \cdot v = v$

$V_{\Lambda_0}(\lambda)$ = wt space of wt λ.

$$ch(V_{\Lambda_0}) = \sum_{\lambda} \dim V_{\Lambda_0}(\lambda)\, e^{\lambda} = \left(e^{\Lambda_0} \sum_{\gamma \in Q} e^{\gamma - \frac{1}{2}\langle \gamma, \gamma \rangle \delta}\right) \cdot \left(\prod_{k=1}^{\infty} \frac{1}{1 - e^{-k\delta}}\right)^{\text{rank}\,\mathfrak{g}}$$

gen. function of partitions.

where Q is the root lattice of $\mathfrak{g}$

... So there ought to be a very explicit construction of V_{Λ_0}.

$\sim$1980 Frenkel–Kac did that

$\left(\prod_{k\geq 1}^{\infty} \frac{1}{1-e^{-k\delta}}\right)^{\operatorname{rank} g}$ is the character of the Heisenberg algebra:

$$(i,j) = \begin{cases} 2 & i=j \\ -1 & i - j \\ 0 & \text{otherwise} \end{cases}$$

$\hat{h}$ = generators $\{h_{in}\}_{\substack{i \text{ root} \\ n \in \mathbb{Z}\setminus 0}}$

w/ $[h_{im}, h_{jm}] = \delta_{m,-n} \cdot n \langle i,j \rangle 1$

$\hat{h}$ has just one irrep, $\mathcal{F}$:

$$\mathcal{F} = \hat{h} \otimes_{\hat{h}^-} \mathbb{C} = \operatorname{Ind}_{\hat{h}^-}^{\hat{h}}(\text{triv})$$

where $\hat{h}^- = \langle h_{in} \rangle_{n<0}$

Set $\deg(h_{in}) = n$ & then

$$ch(\mathcal{F}) = \left(\prod_{n=1}^{\infty} \frac{1}{1-q^n}\right)^{\operatorname{rank}(g)}$$

So V_{Λ_0} looks like one copy of $\mathcal{F}$ for each element of the root lattice.

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