Coleman Lecture 8: Wick's Theorem and Wick Diagrams

December-08-11
12:33 PM

Model 1 Free field interacting with a fixed background.

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - \mu \phi^2 + g \rho \phi$$

ρ vanishes at infinity.

Model 2 Same, but ρ is independent of time.

$$\mathcal{L}_I = g \rho \phi$$

Model 3 ϕ scalar, ψ charged scalar,

$$\mathcal{L}_I = g \phi \psi^* \psi$$

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi - m \psi^* \psi - g \phi \psi^* \psi + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\mu}{2} \phi^2$$

$$T(A(x) B(y)) = :A(x) B(y): + \overline{A(x) B(y)}$$

$$\overline{A(x) B(y)} = \langle 0 | :A(x) B(y): | 0 \rangle$$

$$\overline{\phi(x) \phi(y)} = \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} \frac{1}{p^2 - \mu^2 + i\epsilon}$$

$$\overline{\psi(x) \psi(y)} = \int \ldots$$

Wick's Theorem.

$$T(\phi_1 \ldots \phi_n) = :\phi_1 \ldots \phi_n: + :\text{terms w/ one contraction}: + :\text{all terms w/ two contractions}: + :\ \ldots\ :$$

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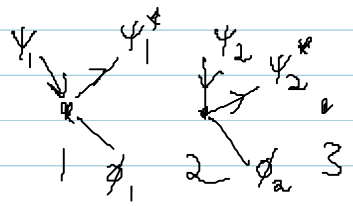
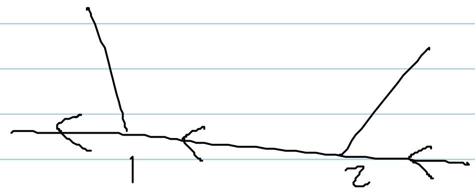
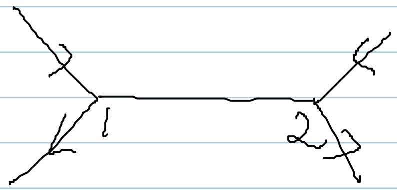
Feynman rules for $H_I = g f(t)\, \psi^* \psi \phi$:



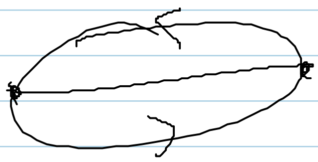
. . . . . then contract.



$$\to \frac{(-ig)^2}{2!} \int :\psi_1^* \psi_1 \phi_1 \psi_2^* \psi_2 \phi_2: \, f(t_1) f(t_2)$$



$$\to \frac{(-ig)^2}{2!} \int dx_1 dx_2\, f(t_1) f(t_2) \cdot :\ \ldots \phi_1 \ldots \phi_2 \ldots\ :$$



$$\to \ldots :\psi_1^* \psi_1 \phi_1 \psi_2^* \psi_2 \phi_2:$$

Combinatorial factors, the order of the automorphism group.

Connected & disconnected diagrams, log & exp.

Back to model I: $H_I = g\phi(x)\rho(x)$