w-Groups

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9:42 AM

Dec 31, 2011: I should see if this should be reformulated in the language of PROPs.

A w-group is the following collection of data:

1. For every pair (H, T) ((heads, tails)) of sets of labels, a set $P(H,T)$ of "elements with heads H & tails T".

2. Obvious head- & tail- renaming operations.

3. An "external product" operation

$$\bullet : P(H_1, T_1) \times P(H_2, T_2) \to P(H_1 \dot\cup H_2, T_1 \dot\cup T_2)$$

4a. For any pair of heads $x, y \in H$ and any "new" head $z \notin H$, a "head multiplication" map

$$m^{xy}_z : P(H,T) \to P(H \cup \{\hat{x}, \hat{y}, z\}, T)$$

4b. For any head z, an "inverse" $S^z : P(H,T) \circlearrowleft$

4c. For any head z and new heads x and y, a "head doubling", or "head coproduct" map

$$\Delta^z_{xy} : P(H,T) \to P(H \cup \{x, y, \hat{z}\}, T)$$

4d,e. unit η_x, co-unit ϵ^x

5. Likewise for tails:

a. $m^z_{xy} : P(H,T) \to P(H, T \cup \{\hat{x}, \hat{y}, z\})$

b. $S_z : P(H,T) \circlearrowleft$

c. $\Delta^{xy}_z : P(H,T) \to P(H, T \cup \{x, y, \hat{z}\})$

6. For any tail x and head y, a "conjugation" or "head on tail action" map:

$$C_x^y : P(H,T) \circlearrowleft$$

All this data subject to all the conditions that are satisfied in the following example:

Example. The Drinfel'd double of a group: G a group, $\mathbb{Z}G$ its group ring, G^* := same but with Abelian structure,

$$P(H,T) := (\mathbb{Z}G)^{\otimes H} \otimes (G^*)^{\otimes T}$$

Definition. a w-group is called **factorizable** if for every $\mu \in P(H,T)$, for every head z and new heads x_1, x_2, and for every splitting of the tails $T = T_1 \dot\cup T_2$, there is an element

$$\nu \in P(H \cup \{x_1, x_2, \hat{z}\}, T)$$

such that

1. $$m_z^{x_1, x_2} \nu = \mu$$

2. $\epsilon_{T_2} \epsilon^{x_1} \nu = \epsilon^{x_1} \nu$ and ...