Maxwell Class in Yael's 1300

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0. Feynman's 8 eqn's.

1. prereq 1: Poincare's Lemma: $d\omega = 0 \Rightarrow \exists \eta$ s.t. $d\eta = \lambda$

2. pre-req 2: Integration by parts: $\int d\omega \wedge \eta = -(-1)^{\omega} \int \omega \wedge d\eta$

3. The Hodge $*$: $\omega \wedge (*\eta) = \langle \omega, \eta \rangle dx_1 \wedge \dots \wedge dx_n$

Example on $\mathbb{R}^4_{txyz}$: $\quad \omega \wedge (*\omega) = \|\omega\|^2 dx_1 \wedge \dots \wedge dx_n$

$*(dx\,dt) = -dy \wedge dz \qquad *(dx\,dy) = -dz\,dt$

$*(dy\,dt) = -dz \wedge dx \qquad *(dy\,dz) = -dx\,dt$

$*(dz\,dt) = -dx\,dy \qquad *(dz\,dx) = -dy\,dt$

4 The least action principle to $F=ma$.

5 $A \in \Omega^1_c(\mathbb{R}^4_{txyz}) \quad J \in \Omega^3_c(\mathbb{R}^4) \quad S(A) = \int \frac{1}{2}\|dA\|^2 dt\,dx\,dy\,dz + J \wedge A$

term prop to ϵ in $S(A+\epsilon B)$ $= 0 = \int \frac{1}{2}(dB \wedge * dA + \underbrace{dA \wedge * dB}_{dB \wedge * dA}) + J \wedge B$

$$= \int B \wedge (d * dA) - B \wedge J = \int B \wedge (d*dA - J)$$

$\Rightarrow \quad d*dA = J$

6. Therefore $dJ = 0$, w/ $F = dA$ $\quad dF = 0 \quad d*F = J$.

7 write $F = E_x dx\,dt + \dots \qquad J = \rho\, dx\,dy\,dz$

$\qquad + B_x dy\,dz + \dots \qquad - j_x dy\,dz\,dt - \dots$

$dJ = 0$: $\left(\frac{\partial \rho}{\partial t} + \frac{\partial j_x}{\partial x} + \dots \right) dt\,dx\,dy\,dz$

so $\frac{\partial \rho}{\partial t} + \operatorname{div} j = 0$

"conservation of charge"

[BTW, charge-current really]

is a 3 form

$dF = 0$: coeff of $dx\,dy\,dz$: $\operatorname{div} B = 0$

coeff of $dx\,dy\,dt$: $-\partial_y E_x + \partial_x E_y + \partial_t B_x = 0$

$\Rightarrow \operatorname{curl} E = -\frac{\partial B}{\partial t}$

$*F = F /.\ \begin{matrix} E \to -B \\ B \to -E \end{matrix}$; so

$d{*F} = J$; $\operatorname{div} E = -\rho$

$\operatorname{curl} B = -\frac{\partial E}{\partial t} + j$

physical interpretation?

8. The ellipticity problem & the Lorentz fix.