Random

October-01-11
7:11 AM

$$e^{A+\epsilon B} = e^A + \epsilon e^A \frac{1-e^{-adA}}{AdA}(B) + \dots = e^A + \epsilon \frac{e^{adA}-1}{adA}(B) e^A + \dots$$

$$\boxplus A^n := \sum_{k=0}^{n} A^k \otimes A^{n-k}$$

$$\boxplus e^A = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k=0}^{n} A^k \otimes A^{n-k} = \sum_{n,m=0}^{\infty} \frac{1}{(n+m)!} A^n \otimes A^m$$

$$e^A \frac{1-e^{-adA}}{adA}(B) = e^{adA}\left(\frac{1-e^{-adA}}{adA}\right)(B)\, e^A =$$

$$= \frac{e^{adA}-1}{adA}(B) \cdot e^A$$

So $$E e^A = e^A \frac{1-e^{-adA}}{adA}(EA) = \frac{e^{adA}-1}{adA}(EA) \cdot e^A$$

$$E(e^x e^y) = e^x x e^y + e^x e^y y = e^x e^y \left(e^{-ad\,y}(x) + y\right)$$

now $[x,y] = c_x y - c_y x = -ad_y(x)$, so

$$= e^x e^y \left(e^{-c_y} \cdot x - \frac{e^{c_y}-1}{c_y} c_x y + y \right)$$

$$E\left(e^{fx+gy}\right) = e^{fx+gy} \frac{1-e^{-ad(fx+gy)}}{ad(fx+gy)} \left((f+Ef)x + (g+Eg)y\right)$$

$$= e^{fx+gy}\left((E+1)f\right.$$

Don't rush to the frontline.