Vogel: The Exceptional Hyperalgeba

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3:22 AM

Invariants: Jones = Kauffman (sl_2), HOMFLYPT (sl_n),
Kauffman Poly ($osp(n)$)

Invariants	Quantum Group	Algebra
Jones = Kauffman	sl_2	T-L
HOMFLYPT	sl_n	Hecke
Kauffman Poly	osp	Birman-Wenzel
A 2-variable poly Z_0	Exceptional: E_6, E_7, E_8, F_4, G_2, Sl_2, Sl_3, D_4, $G(4)$, ?	Z_6 some hyper algebra

We don't really need L, but the category Rep(L).

Conjecture: (Deligne) $\exists$ monoidal category over ... s.t. Rep(L) for any exceptional L is obtained by reduction of coefficients.

<u>Def</u> A Hyper-Algebra is a sequence of algebras A_n together w/ an algebra homomorphism $A_p \otimes A_q \to A_{p+q}$.

Given L, $[\cdot,\cdot]$ (bracket), $\langle \cdot , \cdot \rangle$ (metric)

Rep(L)
$\cup$
only powers of the adjoint = $Rep_0(L)$

There's a description of $Rep_0(L)$ using graphical calculus: Category $\mathcal{D}$: Objects: $[n]$ $n \geq 0$
morphisms generated by

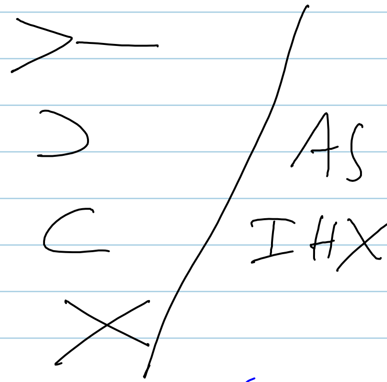
morphisms generated by

$[\,,\,] : L^{\otimes 2} \to L$

$\langle\ \rangle : L^{\otimes 2} \to L^{\otimes 0}$

Casimir : $L^{\otimes 0} \to L^{\otimes 2}$

interchange of L $T : L^{\otimes 2} \to L^{\otimes 2}$



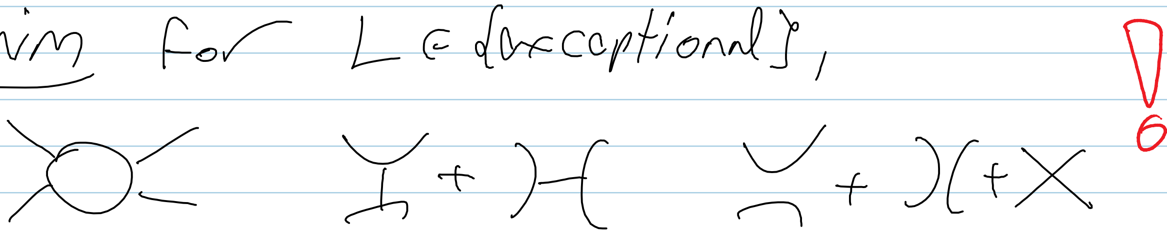
Question: What is $\mathcal{D}_0$ the projectivization of?

There is an obvious monoidal functor

$$\mathcal{D} \longrightarrow \mathrm{Rep}_0(L)$$

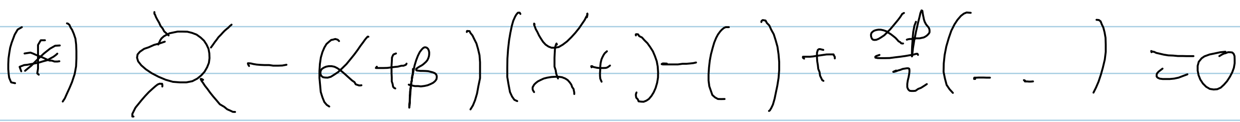
So we are looking for quotients of $\mathcal{D}$

Claim for $L \in \{\text{exceptional}\}$,



these diagrams are linearly related.

In fact, $\exists\ \alpha, \beta$ s.t.



$(*)$ $- (\alpha+\beta)(\ \ +\ \ ) + \frac{\alpha\beta}{2}(\ \ \ ) = 0$

$R = \mathbb{Q}[\alpha, \beta]$ $\quad \mathcal{D}_{excep} := \mathcal{D} \otimes R / (*)$

$L^{\otimes 2} = \Lambda^2 L \oplus S^2 L \qquad S^2 L = k + E$

$I : S^2 L \to S^2 L$ (diagram, not Id!)

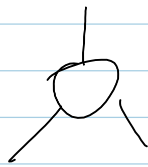
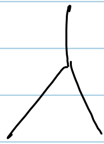
$\Psi : E \to E$

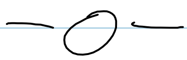
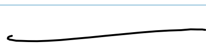
On E, (*) becomes

$$\Psi^2 - (\alpha+\beta)\Psi + \alpha\beta = 0$$

So α, β are eigenvalues of Ψ.

Get also

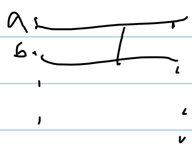
 $= 3(\alpha+\beta)$ 

 $= 6(\alpha+\beta)$ 

Question: Is $\mathrm{End}([0]) \cong \mathbb{R}$?

:

The exceptional hyper algebra E_n is generated by

$S_n \cup$  $= \Psi_{ab}$

modulo the relations

1. $\sigma \in S_n \Rightarrow \sigma \circ \Psi_{ab} = \Psi_{\sigma a, \sigma b} \circ \sigma$
2. $\Psi(a,b) = \Psi(b,a) = (a,b)\Psi(a,b)$
3. $\Psi(ab)^2 - (\alpha+\beta)\Psi(a,b) + \frac{\alpha\beta}{2}(1+(a,b)) = 0$

4. bc

5. 4T

Known: For $n \le 7$ E_n is semi-simple.

$\psi = \sum \psi(a,b)$ is central.

The simple modules for $n \le 7$ are known, & the action of ψ on them.