Joint w/ De-Felice, ...

Main Result. Let F be a sturmian set on the alphabet A, Let X be a finite maximal bifix F-code. Let d be the degree of X. Then X is the basis of a subgroup of index d of G_A, the free group on A.

A code C is the basis of a free submonoid of A^*

bifix = prefix & suffix: Example

$$a^3+a^2ba+a^2b^2+ab+ba^2+baba$$
$$+bab^2+b^2a+b^3$$

The degree of C is the number of decodings on any bi-infinite word.

F is a "Factorial set": closed under taking factors.

A Sturmian set is set F of words, s.t.

* F is closed under reversals

$* \ \forall n \ \exists W_0 \in F \quad |W_0| = n \ \& \ \forall a \in A \ W_0 a \in F$

$\forall W \in F \setminus W_0 \quad |W| = n \Rightarrow \exists! a \in A \ W_0 a \in F$

Assume $|A| = 2$

Fibonacci word: $a \mapsto ab$
$b \mapsto a$

$a \to ab \to aba \to abaab \to abaababa$