Keller: Quiver mutation and quantum dilogarithm identities

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3:15 AM

Context: Quiver mutation is the main combinatorial tool within the definition of cluster algebras:

Lie Theory

↓

quiver mutation
cluster algebras

discrete dynamical systems

alg geom
* stability conditions
* Donaldson-Thomas

today

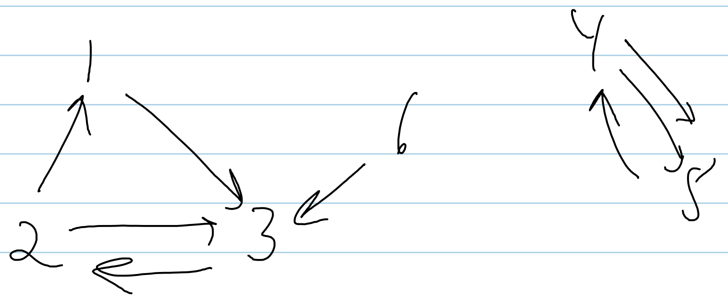
Fock-Goncharov
Nakanishi etc.

Reineke

dilog identities

1. Quiver mutation   2. Q-dilog identities.

Def. A quiver is an oriented graph, w/ multiplicities & self-edges:



Source: No incoming

Sink: No outgoing

For us: quivers are finite quivers w/ no 1 & 2-cycles, and vertices $\{1, \dots, n\}$

Q: A quiver:

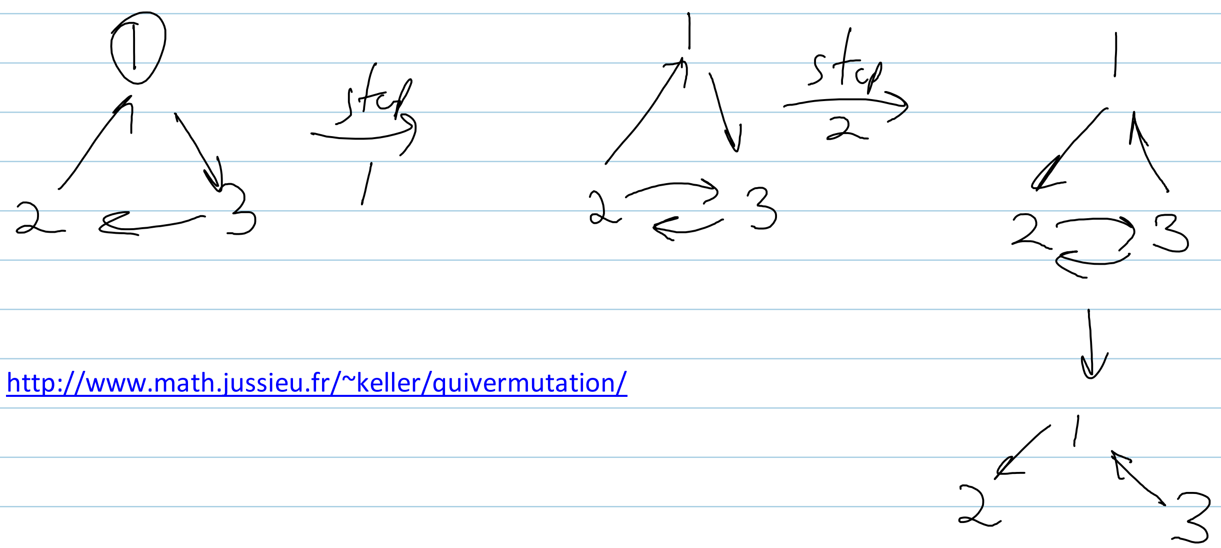
$\mu_j(Q)$ = mutation of Q at j:

1. For each subquiver $i \xrightarrow{\beta} j \xrightarrow{\alpha} k$ add $i \xrightarrow{[\alpha\beta]} k$

2. reverse all arrows incident with j.

3. Remove all 2-cycles that may have been created

Example:



See http://www.math.jussieu.fr/~keller/quivermutation/

2. Quantum dilogarithm identities:

$$\mathbb{E}(y) = 1 + \frac{q^{1/2}}{q-1} y + \dots + \frac{q^{n^2/2} y^n}{(q^n-1)(q^n-q)\cdots(q^n-q^{n-1})} + \dots$$

The pentagon identity: if $y_1 y_2 = q y_2 y_1$ then

$$\mathbb{E}(y_1)\,\mathbb{E}(y_2) = \mathbb{E}(y_2)\,\mathbb{E}(q^{-1/2} y_1 y_2)\,\mathbb{E}(y_1)$$

Faddeev - Kashaev - Volkov (1993)

⋮