At $n=1$, the two are obviously equivalent.
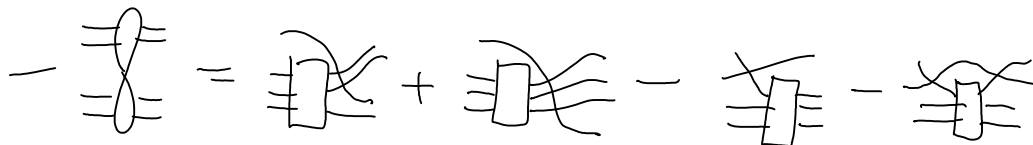
At $n=2$,
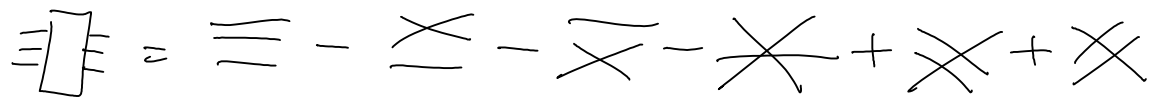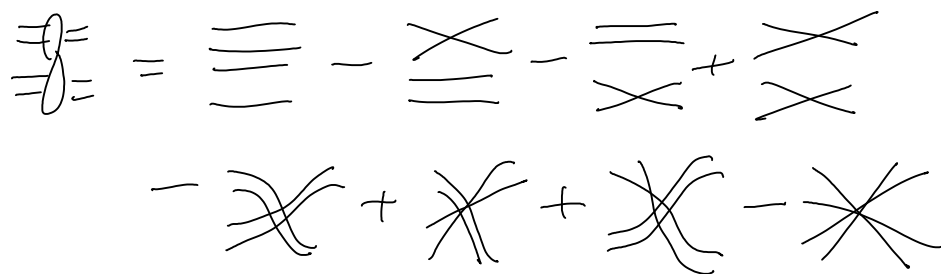
There are 16 ways to put a Λ^3 in S_4.

Mathematica claims:

Joel: Look up "Plethysm".