Pensieve Header: Code for http://katlas.math.toronto.edu/drorbn/bbs/show?shot=Chterental-110630-160804.jpg ; also includes code for counting cycles in a permutation. See also http://katlas.math.toronto.edu/drorbn/AcademicPensieve/2009-07/nb/Immanants.pdf.

```
p = {6, 5, 4, 1, 2, 3}
```

```
{6, 5, 4, 1, 2, 3}
```

```
m = Range[Length[t = p]];
m = MapThread[Min, {m, m[[t]]}];
t = t[[t]]; m = MapThread[Min, {m, m[[t]]}]
```

```
{1, 2, 1, 1, 2, 1}
```

```
CycleCount[p_List] := Module[
  {m, t = p},
  Length[Union[FixedPoint[(
       t = t[[t]];
       MapThread[Min, {#, #[[t]]}]
      ) &,
     MapThread[Min, {Range[Length[p]], p}]
    ]]]
 ]
```

```
CycleCount[{6, 5, 4, 1, 2, 3}]
```

```
2
```

```
(# → CycleCount[#]) & /@ Permutations[Range[4]]
```

```
{{1, 2, 3, 4} → 4, {1, 2, 4, 3} → 3, {1, 3, 2, 4} → 3, {1, 3, 4, 2} → 2,
 {1, 4, 2, 3} → 2, {1, 4, 3, 2} → 3, {2, 1, 3, 4} → 3, {2, 1, 4, 3} → 2, {2, 3, 1, 4} → 2,
 {2, 3, 4, 1} → 1, {2, 4, 1, 3} → 1, {2, 4, 3, 1} → 2, {3, 1, 2, 4} → 2, {3, 1, 4, 2} → 1,
 {3, 2, 1, 4} → 3, {3, 2, 4, 1} → 2, {3, 4, 1, 2} → 2, {3, 4, 2, 1} → 1, {4, 1, 2, 3} → 1,
 {4, 1, 3, 2} → 2, {4, 2, 1, 3} → 2, {4, 2, 3, 1} → 3, {4, 3, 1, 2} → 1, {4, 3, 2, 1} → 2}
```

```
PairingPermutation[a_] := Reverse[Range[2 a]];
PairingPermutation[a_, bs__] := Join[
  PairingPermutation[a],
  2 a + PairingPermutation[bs]
 ]
```

```
PairingPermutation[1, 3, 7]
```

```
{2, 1, 8, 7, 6, 5, 4, 3, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9}
```

```
PairingCycles[as__] := CycleCount[1 + 2 Plus[as] - PairingPermutation[as]] / 2
```

```
PairingCycles[1, 3, 7]
```

```
CycleCont[{21, 22, 15, 16, 17, 18, 19, 20, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}]
```

```
Table[PairingCycles[i, j], {i, 10}, {j, 10}] // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 1 & 1 & 3 & 1 & 1 & 3 & 1 \\ 1 & 2 & 1 & 4 & 1 & 2 & 1 & 4 & 1 & 2 \\ 1 & 1 & 1 & 1 & 5 & 1 & 1 & 1 & 1 & 5 \\ 1 & 2 & 3 & 2 & 1 & 6 & 1 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 7 & 1 & 1 & 1 \\ 1 & 2 & 1 & 4 & 1 & 2 & 1 & 8 & 1 & 2 \\ 1 & 1 & 3 & 1 & 1 & 3 & 1 & 1 & 9 & 1 \\ 1 & 2 & 1 & 2 & 5 & 2 & 1 & 2 & 1 & 10 \end{pmatrix}$$

```
Table[PairingCycles[3, 5, i], {i, 50}]
```

```
{2, 1, 8, 1, 2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1,
 2, 1, 8, 1, 2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2, 1}
```