Quantum Groups Clippings

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3:25 PM

The weirdos are highlighted

E-K1: Global Equations, no symmetry breaking.

page 5:

$$\Phi_{1,2,34}\Phi_{12,3,4} = \Phi_{2,3,4}\Phi_{1,23,4}\Phi_{1,2,3}$$

$$B_{12,3} = \Phi_{3,1,2}B_{1,3}\Phi^{-1}_{1,3,2}B_{2,3}\Phi_{1,2,3},$$

$$B_{1,23} = \Phi^{-1}_{2,3,1}B_{1,3}\Phi_{2,1,3}B_{1,2}\Phi^{-1}_{1,2,3}.$$

Page 11: & 12:

$$R = (J^{op})^{-1}e^{h\Omega/2}J \in U_h(\mathfrak{g})^{\otimes 2},$$

$$R\Delta = \Delta^{op}R,$$

$$(\Delta\otimes 1)(R) = R_{13}R_{23}, (1\otimes\Delta)(R) = R_{13}R_{12}.$$

Page 18: $R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$

Page 10:

$$\Delta(a) = J^{-1}\Delta_0(a)J.$$

$$J = \textstyle\sum_j x_j \otimes y_j,\ x_j, y_j$$

$$Q = \textstyle\sum_j S_0(x_j)y_j$$

$$S(a) = Q^{-1}S_0(a)Q.$$

Etingof-Schiffman (pp 82-3) quote Drinfel'd's "On Almost CoCommutative Hopf Algebras":

$$u = m((S\otimes 1)R^{21})$$

$$S^2(x) = uxu^{-1}$$

From Haviv's Thesis:

Page 48:

$$S^2(x) = uxu^{-1}$$

$$v^2 = uS(u) = S(u)u\,, \quad S(v) = v\,, \quad \epsilon(v) = 1\,,$$

$$\Delta(v) = (v\otimes v)(R^{21}R)^{-1}\,.$$

Page 49:

$$\Delta_F(x) = F\Delta(x)F^{-1}\,, \quad x\in H\,,$$

$$\epsilon_F = \epsilon\,,$$

$$\Phi_F = F^{23}\Delta_2(F)\Phi\Delta_1(F^{-1})(F^{12})^{-1}\,,$$

$$\mathrm{R}_F = F^{21}RF^{-1}\,,$$

Page 85:

$$\Phi\cdot(\Delta\boxtimes\mathrm{id})(J)\cdot J^{12} = (\mathrm{id}\boxtimes\Delta)(J)\cdot J^{23}\,,$$

Page 87:

$$R_{\mathrm{EK}} = (J^{-1})^{21}\cdot\exp(\tfrac{1}{2}\ \vdash\!\dashv\ )\cdot J\,.$$

From Drinfel'd's "On Almost CoCommutative Hopf Algebras":

w/ $B = \{b\in A\otimes A : \forall a\in A,\ [b,\Delta(a)] = 0\}$,

page $R^{21}R \in B$

page 326: Assume quasi-triangular:

$$(\Delta\otimes\mathrm{id})(R) = R^{13}R^{23}\,,$$

$$(\mathrm{id}\otimes\Delta)(R) = R^{13}R^{12}\,,$$

Then

$$(S\otimes\mathrm{id})(R) = R^{-1} = (\mathrm{id}\otimes S^{-1})(R)\,,$$

$$(S\otimes S)(R) = R\,,$$

$$(\varepsilon\otimes\mathrm{id})(R) = 1 = (\mathrm{id}\otimes\varepsilon)(R)\,,$$

$$\Delta(u) = (R^{21}\cdot R)^{-1}\cdot(u\otimes u) = (u\otimes u)\cdot(R^{21}\cdot R)^{-1}\,,$$

$$\Delta(S(u)) = (R^{21}\cdot R)^{-1}\cdot(S(u)\otimes S(u)) = (S(u)\otimes S(u))\cdot(R^{21}\cdot R)^{-1}$$

$$\Delta(z) = (z\otimes z)\cdot(R^{21}\cdot R)^{-2}\,,$$

$$\Delta(g) = g\otimes g\,.$$

COROLLARY. *Suppose, as in Proposition* 3.2, *that* $z = u\cdot S(u) = S(u)\cdot u$ *and* $g = u\cdot S(u)^{-1} = S(u)^{-1}\cdot u$. *Then* $z = (e^{-h\rho}\cdot u)^2$ *and* $g = e^{2h\rho}$.

PROPOSITION 7.2. *Suppose* A *is is any QUE-algebra and* $\mathfrak{g}$ *its classical limit. Then* $h^{-1}(S^2 - \mathrm{id}) \bmod h$ *is a derivation of* $U\mathfrak{g}$ *whose restriction to* $\mathfrak{g}$ *is equal to* $-D/2$, *where* $D\colon \mathfrak{g}\to\mathfrak{g}$ *is the composition of the cocommutator* $\mathfrak{g}\to\mathfrak{g}\otimes\mathfrak{g}$ *and the commutator* $\mathfrak{g}\otimes\mathfrak{g}\to\mathfrak{g}$.

more on that in paper.

A **ribbon Hopf algebra** $(A, m, \Delta, u, \varepsilon, S, \mathcal{R}, \nu)$ is a quasitriangular Hopf algebra which possess an invertible central element ν more commonly known as the ribbon element, such that the following conditions hold:

$$\nu^2 = uS(u),\ S(\nu) = \nu,\ \varepsilon(\nu) = 1$$
$$\Delta(\nu) = (\mathcal{R}_{21}\mathcal{R}_{12})^{-1}(\nu \otimes \nu)$$

where $u = m(S \otimes \mathrm{id})(\mathcal{R}_{21})$. Note that the element *u* exists for any quasitriangular Hopf algebra, and $uS(u)$ must always be central and satisfies $S(uS(u)) = uS(u), \varepsilon(uS(u)) = 1, \Delta(uS(u)) = (\mathcal{R}_{21}\mathcal{R}_{12})^{-2}(uS(u) \otimes uS(u))$, so that all that is required is that it have a central square root with the above properties.

Pasted from <http://en.wikipedia.org/wiki/Ribbon_Hopf_algebra>