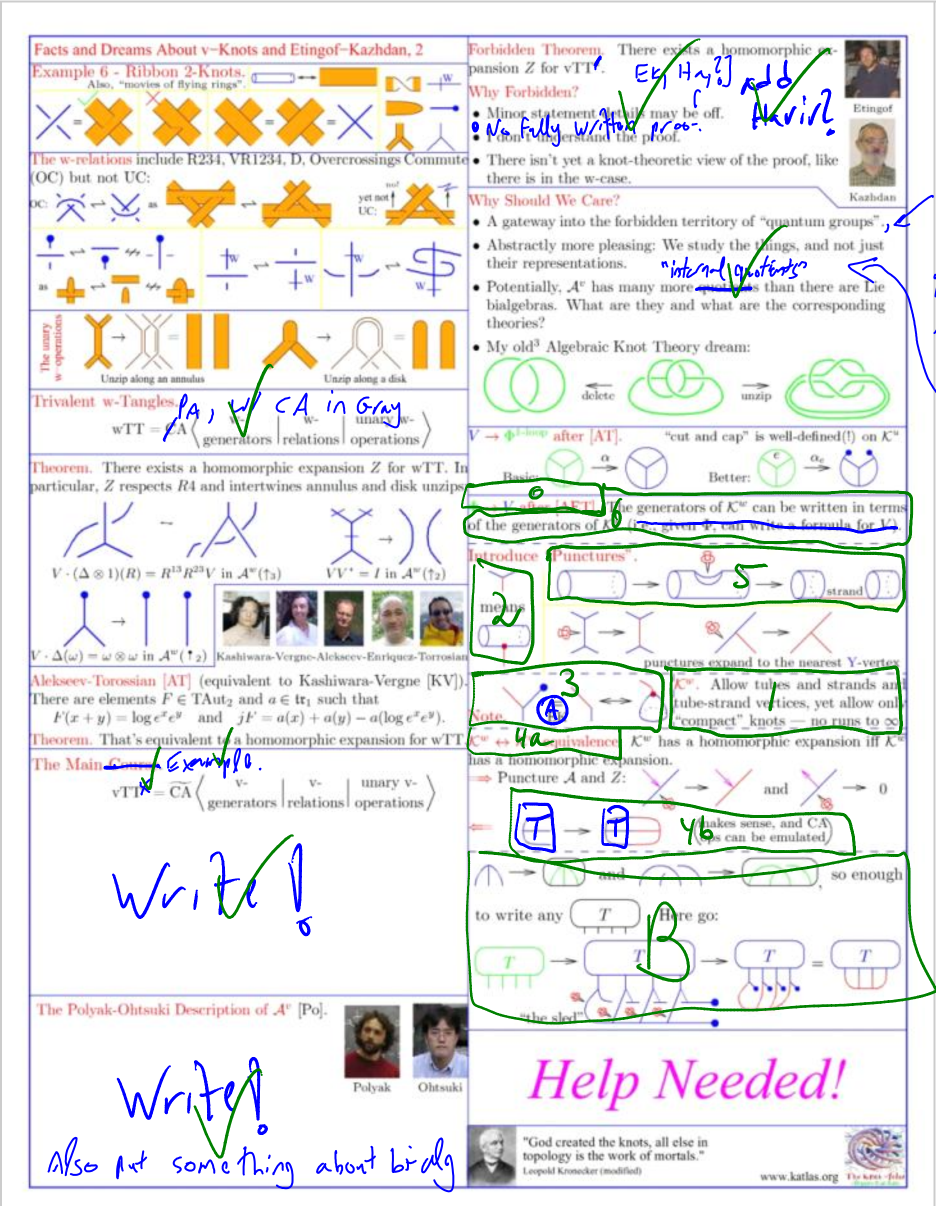
Handout as of May 17

May-17-11
9:48 AM



Facts and Dreams About v–Knots and Etingof–Kazhdan, 2

Example 6 - Ribbon 2-Knots. Also, "movies of flying rings".

The w-relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC:

OC: as ... yet not UC: no!

The unary w-operations: Unzip along an annulus; Unzip along a disk

Trivalent w-Tangles. PA, w/ CA in Gray

wTT = CA ⟨ w-generators | w-relations | unary w-operations ⟩

Theorem. There exists a homomorphic expansion Z for wTT. In particular, Z respects $R4$ and intertwines annulus and disk unzips:

$V\cdot(\Delta\otimes 1)(R) = R^{13}R^{23}V$ in $\mathcal{A}^w(\uparrow_3)$ $VV^* = I$ in $\mathcal{A}^w(\uparrow_2)$

$V\cdot\Delta(\omega) = \omega\otimes\omega$ in $\mathcal{A}^w(\uparrow_2)$

Kashiwara-Vergne-Alekseev-Enriquez-Torrosian

Alekseev-Torossian [AT] (equivalent to Kashiwara-Vergne [KV]). There are elements $F\in \mathrm{TAut}_2$ and $a\in \mathfrak{tr}_1$ such that

$F(x+y) = \log e^x e^y$ and $jF = a(x) + a(y) - a(\log e^x e^y)$.

Theorem. That's equivalent to a homomorphic expansion for wTT.

The Main ~~Course~~ Example.

vTT* = $\widetilde{\mathrm{CA}}$ ⟨ v-generators | v-relations | unary v-operations ⟩

Write!

The Polyak-Ohtsuki Description of $\mathcal{A}^v$ [Po].

Polyak Ohtsuki

Write!

Also put something about bialg

Forbidden Theorem. There exists a homomorphic expansion Z for vTT'. EK, Hay?] add Akriv?

Why Forbidden?

- Minor statement details may be off.
- No fully written proof.
- I don't understand the proof.
- There isn't yet a knot-theoretic view of the proof, like there is in the w-case.

Etingof Kazhdan

Why Should We Care?

- A gateway into the forbidden territory of "quantum groups",
- Abstractly more pleasing: We study the things, and not just their representations.
- Potentially, $\mathcal{A}^v$ has many more ~~quotients~~ "internal quotients" than there are Lie bialgebras. What are they and what are the corresponding theories?
- My old[3] Algebraic Knot Theory dream:

delete unzip

$V \to \Phi^{1\text{-loop}}$ after [AT]. "cut and cap" is well-defined(!) on $\mathcal{K}^u$

Basic: α Better: α_c

0 $\Phi \to V$ after [AET]. 6 The generators of $\mathcal{K}^w$ can be written in terms of the generators of $\mathcal{K}^u$ (i.e., given Φ, can write a formula for V).

Introduce "Punctures". 5

2 means

strand

punctures expand to the nearest Y-vertex

3 Note. Ⓐ

1 $\mathcal{K}^w$. Allow tubes and strands and tube-strand vertices, yet allow only "compact" knots — no runs to ∞

4a $\mathcal{K}^w \leftrightarrow \mathcal{K}^u$ equivalence. $\mathcal{K}^w$ has a homomorphic expansion iff $\mathcal{K}^u$ has a homomorphic expansion.

⇒ Puncture $\mathcal{A}$ and Z: and → 0

⇐ T → T 4b makes sense, and CA ops can be emulated

so enough to write any T. Here go: B

"the sled"

Help Needed!

"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

www.katlas.org The Knot Atlas

and a potential way to harness the power of quantum groups without having to specialize in that dark art.

$\mathcal{A}^v$ is sometimes easier to study than $\mathfrak{g}$: Alexander arises easily from a 2D Lie bialgebra*.

* See [WKO] & Chicago talk.

Ⓐ "$\mathcal{U}(\mathfrak{g})$" ≅ $\mathcal{U}(\mathfrak{g}_+)\otimes M$"; also, reverse that box left-right.

B: with T = ... or ... (or any "classical tangle"), consider

T → the sled story as before.

higher priority.

IF space, include thumbs for axel & IBC!

Footnotes

1. I probably mean "a functor from some fixed "structure multi-category" to the multi-category of sets, extended to formal linear combinations".
2. See my paper [BN1] and my talk/handout/video [BN3].
3. Not so old and not quite written up. Yet see [BN2].

References

[AT] A. Alekseev and C. Torossian, *The Kashiwara-Vergne conjecture and Drinfeld's associators*, arXiv:0802.4300.

[AET] A. Alekseev, B. Enriquez, and C. Torossian, *Drinfeld associators, Braid groups and explicit solutions of the Kashi-waraVergne equations*, Pub. Math. de L'IHES **112-1** (2010) 143–189, arXiv:arXiv:0903.4067.

[BEER] L. Bartholdi, B. Enriquez, P. Etingof, and E. Rains, *Groups and Lie algebras corresponding to the YangBaxter equations*, Jornal of Algebra **305-2** (2006) 742-764, arXiv:math.RA/0509661.

[BN1] D. Bar-Natan, *On Associators and the Grothendieck-Teichmüller Group I*, Selecta Mathematica, New Series **4** (1998) 183–212.

[BN2] D. Bar-Natan, *Algebraic Knot Theory — A Call for Action*, web document, 2006, `http://www.math.toronto.edu/~drorbn/papers/AKT-CFA.html`.

[BN3] D. Bar-Natan, *Braids and the Grothendieck-Teichmüller Group*, talk given in Toronto on January 10, 2011, `http://www.math.toronto.edu/~drorbn/Talks/Toronto-110110/`.

[Dr1,2] V. G. Drinfel'd, *Quasi-Hopf Algebras*, Leningrad Math. J. **1** (1990) 1419–1457 and *On Quasitriangular Quasi-Hopf Algebras and a Group Closely Connected with Gal*($\bar{\mathbb{Q}}/\mathbb{Q}$), Leningrad Math. J. **2** (1991) 829–860.

[EK] P. Etingof and D. Kazhdan, *Quantization of Lie Bialgebras, I*, Selecta Mathematica, New Series **2** (1996) 1–41, arXiv:q-alg/9506005.

[KV] M. Kashiwara and M. Vergne, *The Campbell-Hausdorff Formula and Invariant Hyperfunctions*, Invent. Math. **47** (1978) 249–272.

[Lee] P. Lee, *The Pure Virtual Braid Group is Quadratic*, in preparation.

[Po] M. Polyak, *On the Algebra of Arrow Diagrams*, Let. Math. Phys. **51** (2000) 275–291.

[Th] D. P. Thurston, *The Algebra of Knotted Trivalent Graphs and Turaev's Shadow World*, Geometry & Topology Monographs **4** (2002) 337-362, arXiv:math.GT/0311458.

Plan

1. (8 minutes) The Peter Lee setup for (K, I), "all interesting graded equations arise in this way".
2. (3 minutes) Example: the pure braid group (mention $P\mathcal{B}$, too).
3. (3 minutes) Generalized algebraic structures.
4. (1 minute) Example: quandles.
5. (4 minute) Example: parenthesized braids and horizontal associators.
6. (6 minute) Example: KTGs and non-horizontal associators. ("Bracket rise" arises here).
7. (5 minute) Example: wKO's and the Kashiwara-Vergne equations.
8. (15 minute) vKO's, bi-algebras, E-K, what would it mean to find an expansion, why I care (stronger invariant, more interesting quotients).
9. (5 minute) wKO's, uKO's, and Alekseev-Enriquez-Torrosian.
10. (1 minute) The third page.