Handout as of April 25

April-25-11
11:19 AM

Example 1

Facts and Dreams About v–Knots and Etingof–Kazhdan, 1

Dror Bar–Natan at Swiss Knots 2011

http://www.math.toronto.edu/~drorbn/Talks/SwissKnots-1105/

Abstract. I will describe, to the best of my understanding, the relationship between virtual knots and the Etingof-Kazhdan quantization of Lie bialgebras, and explain why, IMHO, both topologists and algebraists should care. I am not happy yet about the state of my understanding of the subject but I haven't lost hope of achieving happiness, one day.

Abstract Generalities. (K, I): an algebra and an "augmentation ideal" in it. $\hat{K} := \varprojlim K/I^m$ the "I-adic completion". $\operatorname{gr}_I K := \widehat{\bigoplus} I^m/I^{m+1}$ has a product μ, especially, $\mu_{11}\colon (C = I/I^2)^{\otimes 2} \to I^2/I^3$. The "quadratic approximation" $\mathcal{A}_I(K) := \widehat{FC}/\langle \ker \mu_{11} \rangle$ of K surjects using μ on $\operatorname{gr} K$.

Peter Lee

Dream 1?

The Prized Object. A "homomorphic $\mathcal{A}$-expansion": a homomorphic filtered $Z\colon K \to \mathcal{A}$ inducing the identity on $I/I^2 = C$.

Dror's Dream. All interesting graded objects and equations, especially those around quantum groups, arise this way.

Example 2/

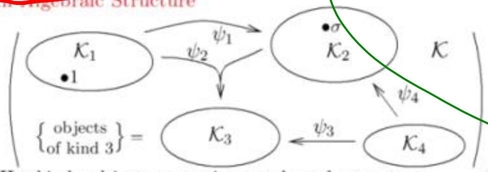
Fact 1

Example 2. For $K = \mathbb{Q}PvB_n$, [ ] Lee shows that a non-homomorphic Z exists. We don't know a homomorphic one.

add some arrow diagram picture.

add a ref.

~~An Algebraic Structure~~



- Has kinds, objects, operations, and maybe constants.
- Must have "the free structure over some generators".
- We always allow formal linear combinations.

All still works!

~~Generalities specialized~~

T. Kohno

$K =$  $\quad I =$ 

$(K/I^{m+1})^\star = (\text{invariants of type } m) =: \mathcal{V}_m$

$(I^m/I^{m+1})^\star = \mathcal{V}_m/\mathcal{V}_{m-1} \quad C = \langle t^{ij} | t^{ij} = t^{ji} \rangle =$ 

$\ker \mu_{11} = \langle [t^{ij}, t^{kl}] = 0 = [t^{ij}, t^{ik} + t^{jk}] \rangle = \langle \text{4T relations} \rangle$

$\mathcal{A} = \begin{pmatrix}\text{horizontal chord dia-}\\ \text{grams mod 4T}\end{pmatrix} =$ 

Z: universal finite type invariant, the Kontsevich integral.

Why Prized? Sizes K and shows it "as big" as $\mathcal{A}$; reduces "topological" questions to quadratic algebra questions; gives life and meaning to questions in graded algebra; universalizes those more than "universal enveloping algebras" and allows for richer quotients.

General Algebraic structures (foot)

Foot: Really, we mean "a functor from some fixed structure multi-category to the multi-category of sets".

Facts and Dreams About v−Knots and Etingof−Kazhdan, 2

To Do.

- Example: quandles.
- Example: parenthesized braids and horizontal associators.
- Example: KTGs and non-horizontal associators. ("bracket rise" arises here).
- Example: wKO's and the Kashiwara-Vergne equations.
- vKO's, bi-algebras, E-K, what would it mean to find an expansion, why I care (stronger invariant, more interesting quotients).
- wKO's, uKO's, and Alekseev-Enriquez-Torrosian.
- The third page.