Handout as of April 20

April-20-11
2:20 PM

Facts and Dreams About v–Knots and Etingof–Kazhdan, 1

Dror Bar-Natan at Swiss Knots 2011

Abstract. I will describe, to the best of my understanding, the relationship between virtual knots and the Etingof-Kazhdan quantization of Lie bialgebras, and explain why, IMHO, both topologists and algebraists should care. I am not happy yet about the state of my understanding of the subject but I haven't lost hope of achieving happiness, one day.

Abstract Generalities. (R, I): an algebra and an "augmentation ideal" in it. $\hat{R} := \varprojlim R/I^m$ the "I-adic completion". $\operatorname{gr}_I R := \bigoplus I^m/I^{m+1}$ has a product μ, especially, $\mu_{11} : (C = I/I^2)^{\otimes 2} \to I^2/I^3$. The "quadratic approximation" $\mathcal{A}_I(R) := TC/\langle \ker \mu_{11} \rangle$ of R surjects using μ on $\operatorname{gr} R$.

Peter Lee

The Prized Object. A "homomorphic $\mathcal{A}$-expansion": a homomorphic filtered $Z : R \to \mathcal{A}$ inducing the identity on $I/I^2 = C$.

Dror's Dream. All interesting graded objects and equations, especially those around quantum groups, arise this way.

Example 2. For $R = \mathbb{Q}PtB_n$, Lee shows that a non-homomorphic Z exists. We don't know a homomorphic one.

Put arrow diagram here?

Generalities Specialized.

$$I = \left\langle \times = \times - \times \right\rangle \qquad R = \left\langle \right\rangle$$

T. Kohno

$(R/I^{m+1})^* =$ (invariants of type m) $\subset \mathcal{V}_m$

$(I^m/I^{m+1})^* = \mathcal{V}_m/\mathcal{V}_{m-1} \qquad C = \langle t^{ij} | t^{ij} = t^{ji} \rangle = \left\langle | \; \right\rangle$

$\ker \mu_{11} = \langle [t^{ij}, t^{kl}] = 0 = [t^{ij}, t^{ik} + t^{jk}] \rangle = \langle 4\text{T relations} \rangle$

$\mathcal{A} = \left(\begin{matrix}\text{horizontal chord dia-}\\ \text{grams mod 4T}\end{matrix}\right) =$ 4T

Z: universal finite type invariant, the Kontsevich integral.

Why Prized? Sizes R and shows it "as big" as $\mathcal{A}$; reduces "topological" questions to quadratic algebra questions; gives life and meaning to questions in graded algebra; universalizes those more than "universal enveloping algebras" and allows for richer quotients.

To Do.

- Generalized algebraic structures.
- Example: quandles.
- Example: parenthesized braids and horizontal associators.
- Example: KTGs and non-horizontal associators. ("bracket rise" arises here).
- Example: wKO's and the Kashiwara-Vergne equations.
- vKO's, bi-algebras, E-K, what would it mean to find an expansion, why I care (stronger invariant, more interesting quotients).
- wKO's, uKO's, and Alekseev-Enriquez-Torossian.
- The third page.

Facts and Dreams About v−Knots and Etingof−Kazhdan, 2

"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

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