

Facts and Dreams About v-Knots and Etingof-Kazhdan, 1

Dror Bar-Natan at Swiss Knots 2011

<http://www.math.toronto.edu/~drorbn/Talks/SwissKnots-1106/>

Abstract. I will describe, to the best of my understanding, the relationship between virtual knots and the Etingof-Kazhdan quantization of Lie bialgebras, and explain why, IMHO, both topologists and algebraists should care. I am not happy yet about the state of my understanding of the subject but I haven't lost hope of achieving happiness, one day.

Abstract Generalities. (R, I) : an algebra and an "augmentation ideal" in it. $\hat{R} := \varprojlim R/I^m$ the " I -adic completion". $\text{gr}_I R := \bigoplus I^m/I^{m+1}$ has a product μ , especially, $\mu_{11}: (V = I/I^2) \rightarrow I^2/I^3$. The "quadratic approximation" $\mathcal{A}_I(R) := TV/(\ker \mu_{11})$ of R surjects using μ on $\text{gr } R$.



Peter Lee

The Prized Object. A "homomorphic \mathcal{A} -expansion": a homomorphic filtered $Z: R \rightarrow \mathcal{A}$ inducing the identity on $I/I^2 = V$.

Dror's Dream. All interesting graded objects and equations, especially those around quantum groups, arise this way.

Example. $R = \mathbb{Q}PuB_n$, the group ring of pure u -braids. $\tau^{ij} = \sigma^{ij} - 1$ generate the augmentation ideal I their images t^{ij} generate V , and $\mathcal{A} = \langle t^{ij} | t^{ij} = t^{ji}, [t^{ij}, t^{kl}] = 0 = [t^{ij}, t^{ik} + t^{jk}] \rangle$. A homomorphic Z exists but is hard to find!

Example 2 For $R = \mathbb{Q}PvB_n$ Lee shows that a non-homomorphic Z exists. We don't know about a homomorphic one.

$I \neq H$ etc. ✓

add a "reading guide?" or "generalities specialized" ✓

why prizes? ✓

X why prizes?

spacing. ✓

$I \neq H$ ✓

X start "commentaries" on page 3.

To Do.

- Example: the pure braid group (mention PvB_n , u) ✓
- Generalized algebraic structures. ✓
- Example: quandles.
- Example: parenthesized braids and horizontal associators.
- Example: KTGs and non-horizontal associators ("Bracket rise" arises here). ✓
- Example: wKO's and the Kashiwara-Vergne equations.
- vKO's, bi-algebras, E-K, what would it mean to find an expansion, why I care (stronger invariant, more interesting quotients).
- wKO's, uKO's, and Alekseev-Enriquez-Torresian.
- The third page.

✓ Kronenker / Katlas.