A Strange Weight System by Duzhin

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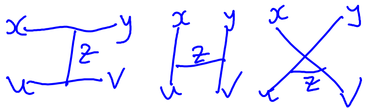
9:04 AM

Date: Tue, 17 May 2011 10:25:52 +0400
From: Sergei Duzhin <...>
To: "D. Bar-Natan" <drorbn@math.toronto.edu>
Subject: kleinian weight systems



Dror,

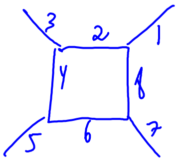
About 10 years ago I told you the construction of a weight system for 1-3-valent graphs starting from a skew-symmetric function of 3 variables F(x,y,z) that satisfies the identity

F(x,y,z)*F(u,v,z)-F(x,u,z)*F(y,v,z)+F(x,v,z)*F(y,u,z)=0

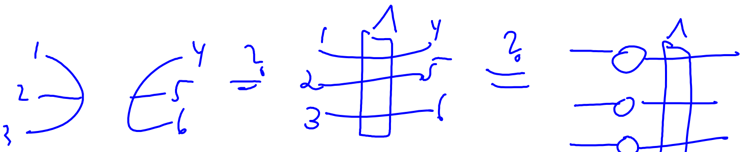
(a typical example is det([f(x),f(y),f(z)],[g(x),g(y),g(z)],[h(x),h(y),h(z)]) , but there are others). To evaluate this weight system on a given diagram, you assign variables to edges, take the product over vertices of the values F(e1,e2,e3) for the three edges meeting at a vertex, and take the average over permutations (of inner and outer vertices separately).

You explained me that the resulting weight system is always equivalent to one coming from sl_2. Now I tried to reconstruct this argument, and I could not:(May I ask you to give a hint, if you still remember that?

Sergei



$$\longrightarrow \sum_{\text{typed perms of } x_1,\dots,x_8} \begin{vmatrix} f(x_2) & g(x_2) & h(x_2) \\ f(x_3) & g(x_3) & h(x_3) \\ f(x_4) & g(x_4) & h(x_4) \end{vmatrix} \cdot \begin{vmatrix} f(x_4) & g(x_4) & h(x_4) \\ f(x_5) & g(x_5) & h(x_5) \\ f(x_6) & g(x_6) & h(x_6) \end{vmatrix} \cdot \ldots$$

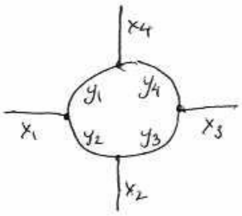


From mail/Duzhin/110520:

Let $F(x,y,z)$ be skew-symmetric and satisfy

$$F(x,y,u)\cdot F(u,v,z) - F(x,u,z)\cdot F(y,v,z) + F(x,v,z)\cdot F(y,u,z) = 0.$$

Then the value of the corresponding weight system on the 4-wheel is obtained as follows:

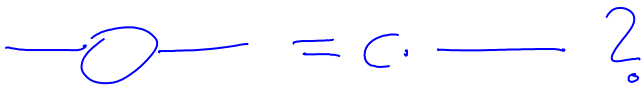


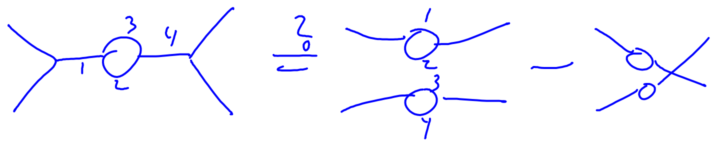
Mark the edges by letters,
then take the product over vertices

$$F(x_1,y_2,y_1)\, F(x_2,y_3,y_2)\, F(x_3,y_4,y_3)\, F(x_4,y_1,y_4).$$

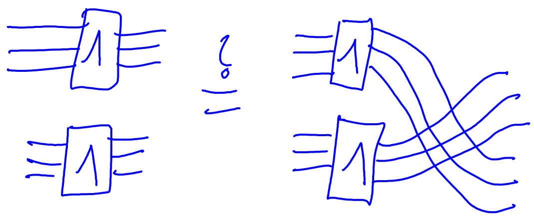
Symmetrize this separately over the permutations of (x_1,x_2,x_3,x_4) ("outer edges") and of (y_1,y_2,y_2,y_3) ("inner edges").

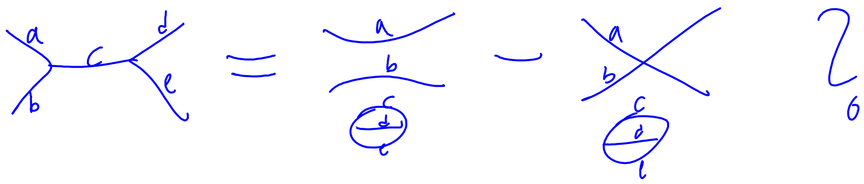
You'll get a polynomial symmetric with respect to $S_{in}\times S_{out}$ where <u>in</u> and <u>out</u> are numbers of inner and outer edges.





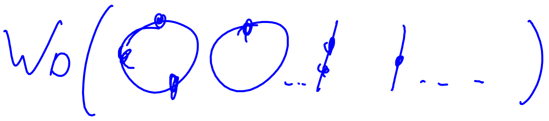
What means 1^3 is 1D?



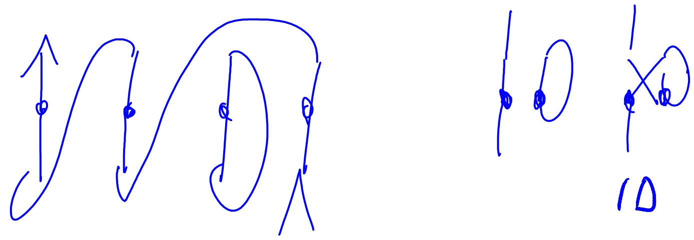


Let W_D be Wouthin, extended with dots.

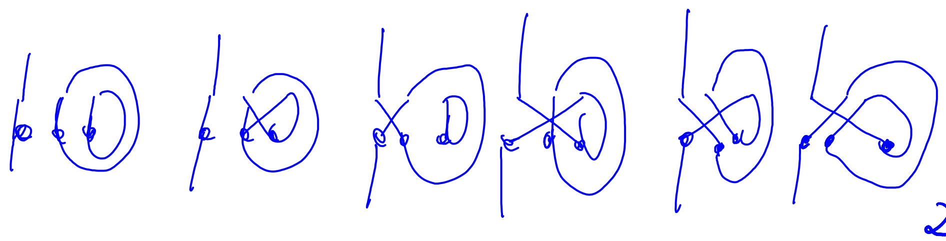
Q. Compute



Are there universal relations between such dotted arcs & circles?



1D



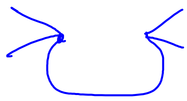
2D

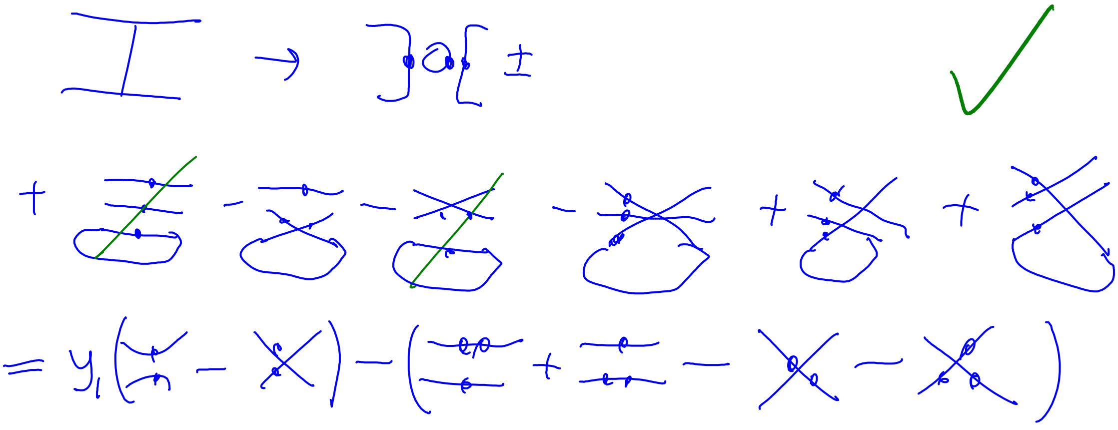
$$x_1y_1^2 - x_1y_2 - 2x_2y_1 + 2x_3 = 0$$

check: if all the x_i are the same and all $y_i = 2$, this is $4x - 2x - 4x + 2x = 0$

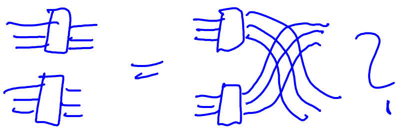
3D: x_4 can be reduced.

Does $Y^2 = \Lambda^3$ implies IHX?



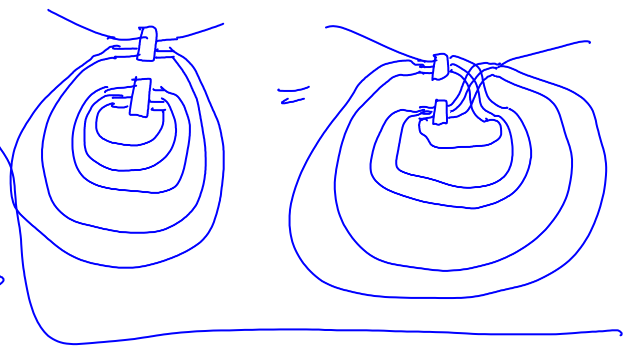


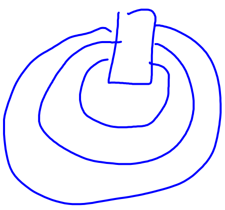
What follows from



$\Lambda^3 = 1D$

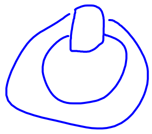
What's the relationship between $\Lambda^3 = 1D$ & $\Lambda^4 = 0$?





$= x^3 - 3x^2 + 2x = 0$

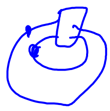
$\Rightarrow x = 0, 1, 2$



$= x^2 - x = 0$

$\Rightarrow x = 0, 1$

yet



$= x_1^2 - x_2 = 0$