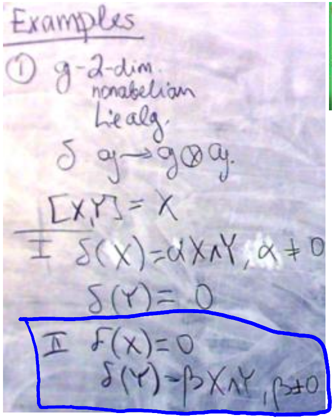
2D BraCobra

May-27-11
4:19 AM



Examples

① g 2-dim. nonabelian Lie alg.

$\delta\ g \to g \otimes g$.

$[X,Y] = X$

I $\delta(X) = \alpha X \wedge Y,\ \alpha \neq 0$
$\delta(Y) = 0$

II $\delta(X) = 0$
$\delta(Y) = \beta X \wedge Y,\ \beta \neq 0$

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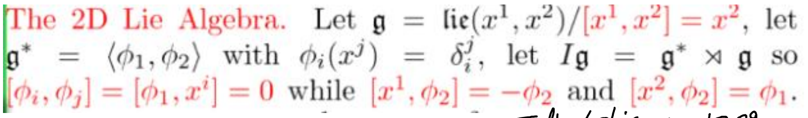

The 2D Lie Algebra. Let $\mathfrak{g} = \mathfrak{lie}(x^1, x^2)/[x^1, x^2] = x^2$, let $\mathfrak{g}^* = \langle \phi_1, \phi_2 \rangle$ with $\phi_i(x^j) = \delta_i^j$, let $I\mathfrak{g} = \mathfrak{g}^* \rtimes \mathfrak{g}$ so $[\phi_i, \phi_j] = [\phi_1, x^i] = 0$ while $[x^1, \phi_2] = -\phi_2$ and $[x^2, \phi_2] = \phi_1$.

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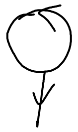
$$g = lie(x^1, x^2)/[x^1, x^2] = x^2$$

$$\delta: \begin{array}{l} x^1 \longrightarrow x^1 \wedge x^2 \\ x^2 \longrightarrow 0 \end{array} \Rightarrow [\phi_1, \phi_2] = \phi_1$$

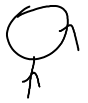
$$[x^1, \phi_1] = \overset{g^*}{0} + \overset{g}{x^2} = x^2$$



$= -x^2$

$$[x^1, \phi_2] = -\phi_2 - x^1$$

$$[x^2, \phi_1] = 0$$



$= \phi_1$

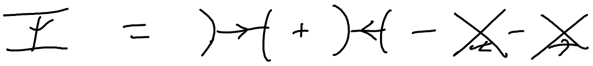
$$[x^2, \phi_2] = \phi_1$$

$$[x, \phi](y) = -\phi([x, y])$$

$$[x, \phi](\lambda) = [\phi, \lambda](x)$$

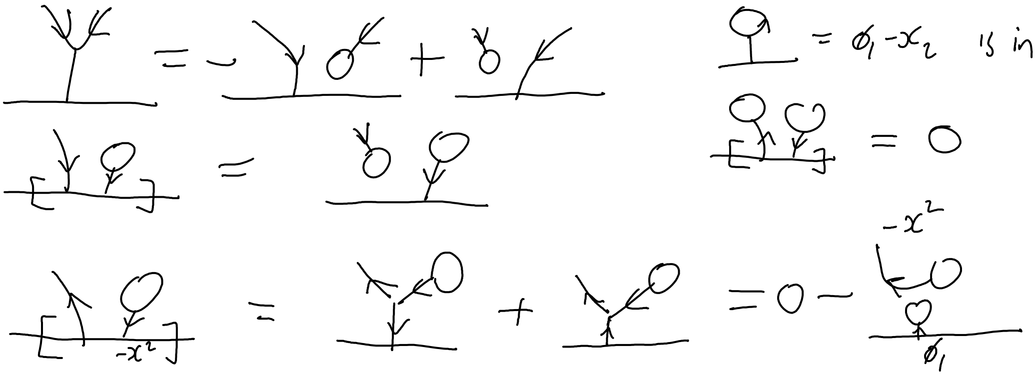
sample Jacobi: $x^2 = [x^1, [x^1, \phi_1]] \overset{?}{=} [[x^1, x^1], \phi_1] + [x^1, [x^1, \phi_1]] = x^2$ ✓

check cocycle: $\delta[x, y] \overset{?}{=} -(1 \otimes \mathrm{ad}\,y - \mathrm{ad}\,y \otimes 1)\delta x + (1 \otimes \mathrm{ad}\,x + \mathrm{ad}\,x \otimes 1)\delta y$

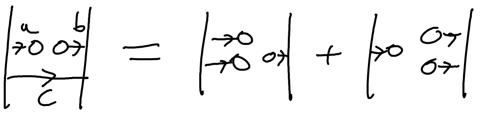


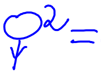
$$x^1 \otimes x^1 \longrightarrow 0 \overset{?}{=} 0 \qquad x^1 \otimes x^2 \rightarrow 0 \overset{?}{=} -(1 \otimes \mathrm{ad}\,x^2 + \mathrm{ad}\,x^2 \otimes 1)(x^1 \wedge x^2)$$

$$= -x^2 \otimes x^2 + x^2 \otimes x^2 = 0$$



$= \phi_1 - x_2$ is invariant!



PS. There is a further quotient, by  $^2 = 0$

as $[\phi_1 \otimes x^1 + \phi_2 \otimes x^2, \phi_1 \otimes (-x^2)] = \phi_1^2 \otimes (-x^2) + \phi_1 \otimes (-x^2)^2$

$$[c, ab] = ab(a+b)$$

$$[c, a] = [\phi_1 \otimes x^1 + \phi_2 \otimes x^2, \phi_1 \otimes 1] = -\phi_1 \otimes x^2 = ab$$

$[c,b] = [\quad - \quad , 1\otimes(-x^2)] = -\emptyset,\otimes x^2 = ab$

$[c,ab] = abb + aab = ab(a+b)$

$[c,[c,ab]] = abb(a+b) + aab(a+b) + 2abab =$

$= ab\left((a+b)^2 + 2ab\right) = ab(a^2+4ab+b^2)$

$\xrightarrow{ad_c} \quad ab(a+b)(a^2+4ab+b^2) + ab(2a+4(a+b)+2b)ab$

$= ab(a+b)(a^2+b^2+10ab)$

$\xrightarrow{ad_c}$

$s = a+b, \; p = ab \qquad [c,s] = 2p \quad [c,p] = s\cdot p$

$p \xrightarrow{ad_c} sp \xrightarrow{ad_c} 2p^2 + s^2p = p(s^2+2p) \xrightarrow{ad_c} sp(s^2+2p) + p(4sp+2sp)$

$= ps(s^2+8p) \xrightarrow{ad_c} sps(s^2+8p) + 2p^2(s^2+8p) + ps(4sp+8sp)$

$=$

$\gamma_1(t) = e^{t\,ad_c}(s) \qquad \gamma_1(0) = s$

$\gamma_2(t) = e^{t\,ad_c}(p) \qquad \gamma_2(0) = p$

$\dot{\gamma}_1(t) = ad_c(\gamma_1(t)) = 2p\frac{\partial}{\partial s}\gamma_1(t) + s\cdot p\cdot\frac{\partial}{\partial p}\gamma_1(s)$

Compare Mathematica / Computing_adc and http://oeis.org/wiki/Eulerian_numbers,_triangle_of and http://en.wikipedia.org/wiki/Eulerian_numbers!

Also: $[c,a] = ab \quad [c,b] = ab$

What if $ad_c(a) = a^2$?

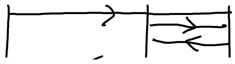
$Da = a^2$

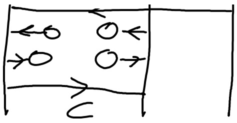
$Da^2 = 2a\cdot a^2 = 2a^3$

$Da^3 = 3a^2a^2 = 3a^4 \ldots$

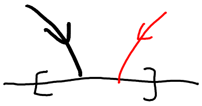
$e^{tD}(a) = \sum \frac{t^nD^n}{n!}a$

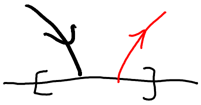
$= a + a^2 + \frac{2a^3}{2!} + \frac{3a^4}{3!} = a + a^2e^a$

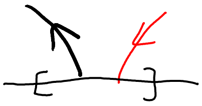
With  I need to know the commutation

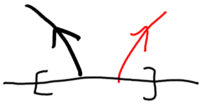


relations of C & of $e^{\pm C}$ with all other creatures.

 =

 =

 =

 =